

[1] En $\mathbb{Z}/(m)$, probar que si $[a] \neq 0$:

a) si $\text{m.c.d.}(a, m) = 1 \Rightarrow [a]$ tiene inverso respecto al producto

b) si $\text{m.c.d.}(a, m) \neq 1 \Rightarrow [a]$ es un divisor de cero

Solución:

[a] $\text{m.c.d.}(a, m) = 1 \Leftrightarrow \exists h, k \in \mathbb{Z} \ni ha + km = 1 \Rightarrow$

$$[1] = [ha] + [km] = [h][a] + [k][\underset{0}{m}] = [h][a]$$

$$\Rightarrow [a]^{-1} = [h]$$

[b] $\text{m.c.d.}(a, m) = d \neq 1 \Rightarrow \exists a', m' \neq 0 \ni a = a'd, m = m'd$

$$\Rightarrow m'a = m'a'd = ma'$$

$$\Rightarrow [m'a] = [ma'] = [m][a'] = 0 \cdot [a'] = 0$$

[2] Encontrar los divisores de cero de $\mathbb{Z}/(24)$

Soluc.

Aplicando 1 b) anterior, resulta que los divisores de cero de $\mathbb{Z}/(24)$ son los $a \in \mathbb{Z} \ni$

$$\text{m.c.d.}(a, 24) \neq 1$$

$$\Rightarrow x \in \{2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22\}$$