Seventeenth International Conference Zaragoza-Pau on Mathematics and its Applications
Jaca, September 4–6th 2024

The Cauchy-Stokes problem

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SUMMARY

Inverse problems associated with elliptic equations or systems refere to the challenging task of determining unknown parameters, geometry or boundary conditions. The Cauchy problem has arisen from many practical contexts, such as non-destructive testing, electrocardiography, electroencephalography and hydro-geology. A tremendous body of literature has referred back to the leading work of Hadamard [4]. In his work, Hadamard highlighted the ill-posedness of such an inverse problem which enormously increases the difficulties of its numerical treatment. In this presentation, we discuss a family of methods that relies on a fictitious domain decomposition formulation. we will also discuss, when dealing with the Stokes problem the sub-Cauchy problem : we reduce the accessible boundary data to consider only the velocity and the shear stress. To address these issues, we propose new Lagrange multiplier methods. These methods involve recasting the problem in terms of interfacial equations, equalizing two solutions of the sub-Cauchy–Stokes problem using matching conditions defined on the inaccessible boundary. The matching process relies on second-order conditions, and the types of interfacial equations depend on the equations used to match the unknown values on the inaccessible boundary. We solve the resulting interfacial problems iteratively, optimizing coefficients to improve convergence rates. Our comprehensive analysis demonstrates the effectiveness and performance of the proposed approaches. we also adress a new method based on Nash game theory.

Keywords: Inverse problems, Cauchy problem, Stokes equations, Nash game, Sub-Cauchy–Stokes, Uniqueness, Ventcell boundary conditions, Alternating method, Shear stress...

AMS Classification: 65F08, 35R30

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