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Finite-volume approximation of a stochastic Allen-Cahn equation with constraint

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SUMMARY

We are interested in the convergence analysis of a finite-volume scheme for the following Allen-Cahn problem perturbed by a multiplicative Itô noise : finding $0 \leq u \leq 1$ satisfying

$$\begin{cases} du + \big(\partial I_{[0,1]}(u) - \Delta u\big)\, dt & \ni & (k(u) + f)\, dt + g(u) dW & in\ \Omega \times D \times (0,T) \\ u(\omega, x, t = 0) & = & u_0(x) & \omega \in \Omega, x \in D, \\ \nabla u.\mathbf{n} & = & 0 & in\ \Omega \times \partial D \times (0,T), \end{cases} \tag{1}$$

where $T > 0$, D is a smooth bounded domain of $\mathbb{R}^2$, $(\Omega, \mathcal{F}, \mathbb{P})$ is a probability space endowed with a right-continuous, complete filtration $(\mathcal{F}_t)_{t\geq 0}$ and $(W(t))_{t\geq 0}$ is a standard, one-dimensional Brownian motion with respect to $(\mathcal{F}_t)_{t\geq 0}$ on $(\Omega, \mathcal{F}, \mathbb{P})$. In a former work, [1], the authors proved the well-posedness of Problem (1) by using a monotone regularization $(\psi_\epsilon)_{\epsilon>0}$ "à la Moreau-Yosida" of the subdifferential $\partial I_{[0,1]}$ and by passing to the limit with respect to ϵ on the "ϵ"-regularized version of Problem (1). Our strategy is to build a time and space discretization scheme on the "ϵ"-regularized version of Problem (1) studied by [1], and to pass to the limit simultaneously with respect to ϵ and the time and space steps denoted respectively by Δt and Δx. Combining a semi-implicit temporal discretization with a Two-Point Flux Approximation (TPFA) scheme for the spatial's one, we are able to prove the convergence of such a "$(\epsilon, \Delta t, \Delta x)$" scheme towards the unique weak solution of Problem (1) by assuming $\Delta t = O(\epsilon^4)$ and the following assumptions on the data:

H_1: $u_0 \in L^2(\Omega, H^1(D))$ and $0 \leq u_0(\omega, x) \leq 1$ for almost all $(\omega, x) \in \Omega \times D$.

H_2: $k, g : \mathbb{R} \to \mathbb{R}$ are Lipschitz-continuous functions with $k(0) = 0$ and $\text{supp}(g) \subset (0, 1)$.

H_3: f is a predictable process belonging to $L^2((0,T) \times \Omega, L^2(D))$.

Keywords: Stochastic PDE, differential inclusion, monotone operators, Itô integral, multiplicative noise, predictable processes, finite-volume method, TPFA scheme,...

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References

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