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Histopolation via mock-Chebyshev points

Ludovico Bruni Bruno[1], **Francesco Dell'Accio**[2], **Wolfgang Erb**[1], **Federico Nudo**[1]

SUMMARY

In many numerical applications, we often encounter phenomena for which we only have measurements at a set of equally spaced points. When using standard polynomial interpolation to approximate such phenomena, the results can be highly inaccurate due to the Runge phenomenon. Several techniques have been introduced to mitigate this issue, for example, the mock-Chebyshev subset interpolation and the constrained mock-Chebyshev least squares approximation. The excellent accuracy achieved by these approximations has led to their widespread use in various applications. Motivated by the success of these techniques in the classical polynomial interpolation, we aim to extend the mock-Chebyshev subset interpolation and the constrained mock-Chebyshev least squares approximation to the case of interpolation on segments. Specifically, we present three detailed generalizations of these methods in this context. The interpolation on segments is a mathematical technique used to approximate a function f over a specific interval $I = [a, b]$. It offers a distinct approach compared to classical polynomial interpolation. While the classical polynomial interpolation relies solely on function evaluations at specific points, the interpolation on segments leverages information about the integral of the function f over a set of subintervals of the interval I. This difference is crucial because the interpolation on segments only requires the function to be essentially bounded, a less restrictive condition than the continuity required for classical polynomial interpolation. We demonstrate that two of these three new methods achieve optimal growth rates for the Lebesgue constant of the corresponding Vandermonde matrix. Specifically, one method boasts logarithmic growth, while another exhibits growth between logarithmic and square-root. Finally, we compare the performance of these new approximation techniques through various numerical experiments.

Keywords: Polynomial interpolation on segments, mock-Chebyshev nodes

AMS Classification: 41A05, 41A10

References

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[1]Department of Mathematics Tullio Levi Civita
University of Padova
email: ludovico.brunibruno@unipd.it, wolfgang.erb@unipd.it, federico.nudo@unipd.it

[2]Department of Mathematics and Computer Science
University of Calabria
email: francesco.dellaccio@unica.it