Seventeenth International Conference Zaragoza-Pau on Mathematics and its Applications
Jaca, September 4–6th 2024

Shape optimization with discontinuities

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SUMMARY

This presentation focuses on a shape optimization method with non-standard transmission conditions on the interface that involves discontinuities and the Laplace-Beltrami operator. From the mathematical point of view, this work can be seen as a generalization of [2]. We consider a framework of two materials that are separated by a thin layer, where doing an asymptotic expansion of order one with respect to the width of the thin layer, it appears a kind of Ventcell condition. In particular, we consider two applications: a heat insulation problem and a heat exchanger one (similar to [1]). Thanks to this approximation, the multi-physics problem is reduced to a weak coupled problem between the Navier-Stokes equations (in laminar regime) for the fluid part and the convection-diffusion equation to describe the temperature. In order to solve these equations, we first have to talk about the finite element methods, since the convection-diffusion has non-standard transmission condition. Because of the discontinuites, the functional spaces involved in the variational formulation are the so-called *Broken Sobolev Spaces*, that allows to work with jumps and mean on the interface. We propose a Nitsche extended finite element approach [3] and we compare it with a Galerkin Discontinuous method. The coercivity, stability and error estimation are proven and verified numerically. Afterward, we come back to the shape optimization problem of interest, where we characterize the shape derivatives and we perform 3D numerical simulations.

Keywords: Shape optimization, Navier-Stokes equations, heat exchangers, Nitsche method, Ventcel boundary condition, asymptotic model, discontinuous Galerkin method.

AMS Classification: 74P10, 76B75, 74F05, 65N15, 65N30, 35S15, 35C20

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