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High-order Melnikov functions through normalisation

Martha Alvarez[1], **Jesús F. Palacián**[2], **Patricia Yanguas**[3]

SUMMARY

We introduce a methodology in order to get the high–order terms of the stable and unstable manifolds associated to a periodic orbit for a Hamiltonian system of one and a half degrees of freedom. The main goal is the analysis of the transversality between both manifolds even when the first term in the Melnikov function does not provide this information, either because it is zero or it is non-dominant. Assuming the Hamiltonian system has fast oscillations we apply suitable normalisation techniques to push the time-dependence of the Hamiltonian function to a high order, making it exponentially small under analyticity conditions of the function. In the normalised system we get the parameterisations of the invariant manifolds readily up to exponentially small terms. We return to the initial system using the forward Lie transformation. In case that the distance of the stable and unstable manifolds be exponentially small some times it is possible to obtain the normal form Hamiltonian with a remainder smaller than the distance between the manifolds. We illustrate the technique by means of two examples: a Duffing oscillator with a time-dependent small perturbation and a restricted isosceles three body problem where the primaries move along an elliptic orbit.

Keywords: Restricted N-body problem, invariant manifolds at infinity, McGehee's coordinates, transversality of manifolds

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[1]Dpto. Matemáticas,
UAM–Iztapalapa, 09310 Iztapalapa, Mexico City, Mexico
email: mar@xanum.uam.mx

[2]Departamento de Estadística, Informática y Matemáticas
Universidad Pública de Navarra, 31006 Pamplona, Spain
email: palacian@unavarra.es

[3]Departamento de Estadística, Informática y Matemáticas
Universidad Pública de Navarra
email: yanguas@unavarra.es