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A quantile regression model for a bounded response variable having the reflected LEEG distribution†

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SUMMARY

A number of continuous probability distributions with bounded support have been introduced in the statistical literature in the last decades. In particular, Jodr and Jimnez-Gamero [3] introduced the LEEG distribution from the extended exponential-geometric distribution (cf. [1]). Here, a new model with bounded support is proposed which corresponds to the reflected LEEG (RLEEG) distribution and has cumulative distribution function $F(x) = (1-(1-x)^\alpha)/(1+\beta(1-x)^\alpha)$, $0 < x < 1$, with parameters $\alpha > 0$ and $\beta > -1$.

A characterization of the RLEEG distribution reads as follows. Let N and M be random variables having geometric distribution with parameters $1/(1+\beta)$ with $\beta > 0$ and $(1+\beta)$ with $\beta \in (-1, 0)$, respectively. Let $T_1, T_2, \ldots$ be iid random variables having reflected power function distribution with parameter $\alpha > 0$, that is, its cdf is $F_{T_i}(t; \alpha) = 1-(1-t)^\alpha$, $0 < t < 1$. Assume that N and M are independent of T_i for $i = 1, 2, \ldots$. Then, $V = \max\{T_1, \ldots, T_N\}$ has RLEEG distribution with $\alpha > 0$ and $\beta > 0$ whereas $W = \min\{T_1, \ldots, T_M\}$ has RLEEG distribution with $\alpha > 0$ and $\beta \in (-1, 0)$. Moreover, closed-form expressions can be given for the quantile function $F^{-1}(u; \alpha, \beta) = 1 - [(1-u)/(1+\beta\, u)]^{1/\alpha}$, $0 < u < 1$, and for the moments $E[X^k] = (1+\beta)\alpha^{-1} \sum_{j=1}^{k} (-1)^{j+1} \binom{k}{j} j\, \Phi(-\beta, 1, 1+j/\alpha)$ for $k = 1, 2\ldots$ where Φ denotes the Lerch transcendent function.

Additionally, a quantile regression model is introduced assuming that the response variable has RLEEG distribution and is related to a set of covariates through a regression structure. Regression models commonly express the mean of a response variable as a function of a set of covariates. However, the mean of the RLEEG distribution is an involved expression which makes difficult to build that kind of model. By contrast, the expression of the quantiles is simple and a quantile regression model can be developed relating the median response with the covariates. A real data application illustrates the suitability of the proposed regression model, which is compared to the beta (cf. [2]) and LEEG (cf. [3]) regression models.

Keywords: Exponential-geometric distribution, bounded support, regression quantile model

AMS Classification: 60E05, 62J02

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