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A posteriori error analysis based on equilibrated fluxes for interface problem with CutFEM

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SUMMARY

The importance of reconstructing conservative fluxes from primal finite element solutions of a boundary value problem is widely recognized in the literature. One appli- cation of such fluxes is in a posteriori error analysis, where the difference between the numerical flux and the reconstructed flux provides a reliable error indicator, which can then be utilized in adaptive mesh refinement.

We focus on the local reconstruction of conservative fluxes in the finite element space of Raviart-Thomas for elliptic interface problems with discontinuous coefficients. For this purpose, we have chosen the approach developed in 2016 by R. Becker , D. Capatina and R. Luce for the Poisson equation, which offers a unified framework for various finite element methods and does not require to solve any mixed problem. We have addressed, both from a theoretical and a numerical point of view, the case of meshes following the interface in a previous work. We considered conforming as well as non-conforming finite element approximations of any degree, and we focused on the robustness of the reconstruction with respect to the coefficients.

In this presentation, we consider a diffusion problem in the presence of an inter- face non-aligned with the mesh. The diffusion coefficients as well as the physical flux are discontinuous across the interface. The transmission conditions are handled using the Nitsche method, and the numerical treatment of the interface is carried out by the CutFEM method. Following our previous work, we first reconstruct a flux on each subdomain. However, the global flux defined by restriction does not satisfy the trans- mission condition on the interface. Regarding the conservation property on the cut elements, we proved that the divergence of each flux is equal to an extension of the given data. To address these shortcomings, we subsequently reconstruct a second flux in the immersed Raviart-Thomas space of lowest degree, recently introduced by J. HI in 2022. This new flux is naturally conservative and satisfies the jump condition on the interface, but it is only piecewise Raviart-Thomas on the cut elements; further- more, its normal traces are weakly continuous across the cut edges. Finally, we propose another flux reconstruction in the Raviart-Thomas space on the whole domain, which is globally H(div) when the jump of the physical flux on the interface is zero. We then use this flux to define an a posteriori error estimator and prove its robustness, reliability and efficiency.

Keywords: Interface problem, cutFEM, a posteriori error analysis, flux reconstruction . . .

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