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Asymptotic error analysis for stochastic gradient optimization schemes with first and second order modified equations

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SUMMARY

Our study explores the relationship between the gradient algorithm and the gradient flow field, aiming to understand and enhance discrete optimization schemes. By analyzing the long-term behavior of the stochastic gradient algorithm through either a ordinary or stochastic differential equation, we employ Lyapunov functionals to study the continuous system and estimate the systematic error between the discrete scheme and the continuous system. Our results provide uniform error estimates over time, and we also derive estimates for a modified equation where the approximation error is of order two, thus improving the convergence accuracy of the discrete scheme. In conclusion, our study offers valuable tools for enhancing the understanding and efficiency of stochastic gradient algorithms in optimization.

Keywords: Optimization, Stochastic, Scheme

AMS Classification: 49M25, 60H20

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