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SPECTRAL PATTERNS OF THE IBEX 35 IN AN INTERNATIONAL CONTEXT

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Abstract. The financial crisis of the period 2008-2014 has specially hit the countries of Southern Europe, and particularly Spain. This circumstance has caused an anomalous behavior of the stock market of our country. Our purpose is to search if the reference index of the Spanish market IBEX 35 owns some distinguishing features with respect to other international indices, from a numerical point of view. For it, the first goal is the improvement of the oscillators currently employed, by means of procedures of interpolation and approximation. The current techniques are complemented with integral parameters, coming from the implementation of trend curves. The functions obtained help to determine the basic behavior of the index, reducing the difficulty of the daily unpredictability of the quotes. The second objective is a numerical and spectral comparison of the evolution of the IBEX 35 with respect to other international indicators.

Keywords: Stock indices, IBEX 35, Hjorth parameters, Power spectrum, Fourier analysis.
AMS classification: 65D05, 42A10, 37M10, 65D10.

§1. Introduction

A stock market index is a mathematical weighted sum of some values that are listed in the same market. Since the creation of the first one in 1884, the stock market indices have been used as an economic and financial activity measure of a particular sector of the market or a country.

The precursors of the establishment of stock indices were Charles Henry Dow (1851-1902) and Edward David Jones (1856-1920) who created the first average of stock values within The Wall Street Journal financial newspaper with the purpose of measure the economic welfare of the United States of America. Subsequently, in 1887, the Dow Jones Industrial Average (DJIA) made up of industrial companies and the Dow Jones Railroad Average formed by railway companies were created. Later, the Dow Jones Transportation Average (DJTA) would group together values of the transport sector ([6]).

Three international stock market indices have been analyzed in this article from a spectral perspective, one is American (Standard & Poor's 500), the second is Japanese (Nikkei 225) and the last one is Spanish (IBEX 35).

Standard & Poor's 500, also known as S&P 500, is an index based on the market capitalization of the 500 largest companies in the NYSE (New York Stock Exchange) and NASDAQ (National Association of Securities Dealers Automated Quotation). While originally the average of each company was capitalization-weighted, since 2005 a transition to a float-weighted system was performed, ie, values are weighted only by the price of the actions that are susceptible to be traded, ignoring those belonging to the group itself, government, long-term investors, etc ([1]). It differs from other major American indices like the Dow Jones Industrial Average or the Nasdaq in the weighting method and its components. The S&P 500 is considered the most representative index of the American market situation ([5]), in fact it is considered representative of 70% of it.

Nikkei 225, commonly referred to as Nikkei index, is the most used in the Japanese market and is composed of the 225 most liquid values in the Tokyo Stock Exchange. It is similar to the Dow Jones Industrial Average but reflects the entire market, without focusing on any sector. One of the characteristics of the Nikkei 225 index is that their values are weighted by price and not by their capitalization.

IBEX 35, established in 1992, is made up of the 35 most liquid companies listed on the four Spanish stock exchanges (Madrid, Barcelona, Bilbao and Valencia). IBEX 35's weighting is based on the market capitalization of the companies. Currently, those with higher weight are Banco Santander, Telefónica, BBVA and Inditex representing more than 50% of the average.

In the following graphs, the price of each stock index analyzed in the work is shown from January 1, 2000 until December 31, 2013.

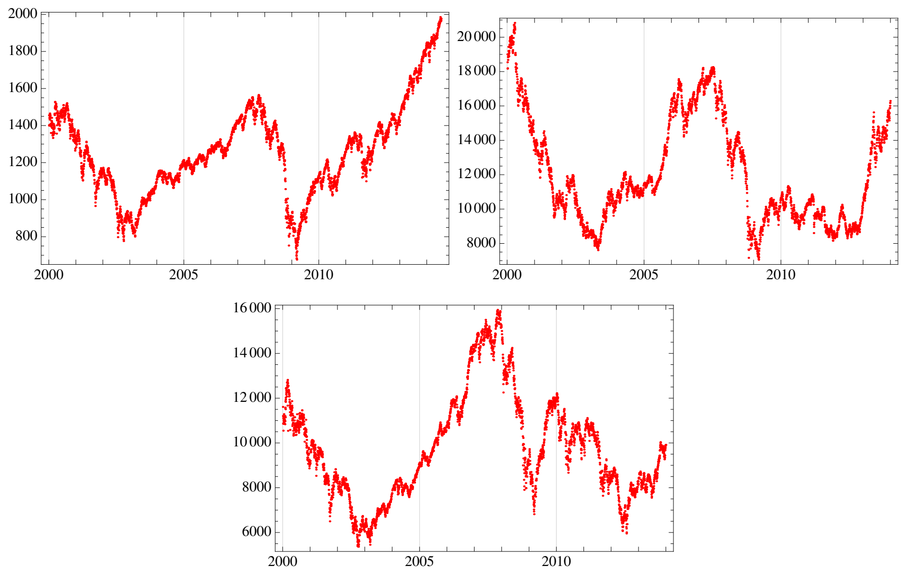

Fig. 1. Evolution of the daily closing price of the S&P, Nikkei and IBEX from year 2000 to 2013 (from left to right and top to bottom)

The widespread growth since 2003 is the effect (more marked in Spain) of the property boom and mainly the effect of subprime mortgages that increased prices of the major indices until the advent of the 2008 financial crisis. Spain suffered in 2010 the so called Euro crisis due mostly to the increase in sovereign debt, which created mistrust among investors while the rating agencies reduced the rating of eurozone bonds. Japan entered economic recession after the financial crisis of 2008. Later changes in financial policies of the Japanese government caused a rebound in the price of the Nikkei index and thus the entire Japanese economy.

In this work we describe numerical patterns, from a spectral view, of three international stock indices, omitting deliberately any economic or casual argument. Our team has implemented a quantification of the stock records using Hjorth parameters, which describe the signal in the time domain and/or the frequency and used them for each of the indices, beginning from approximation curves. We look for descriptors indicating crisis and, in general, strong variations in closing prices. With these quantifiers we wish to analyze a possible relationship between the different indices in order to find correlations in the pre-crisis and the crisis periods as well.

§2. Spectral Analysis

2.1. Approximation curves

In order to obtain the Fourier coefficients necessary for the computation of Hjorth parameters, we begin with an approximation curve of the original signal, that is to say, of the paired data (t_i, x_i), where x_i is the closing price of every index corresponding to day t_i. As first approximant of the signal, we chose an interpolation method that best fitted the records. Polynomial interpolation was excluded because it produced large errors. The methods considered were linear and natural cubic spline, due to their simplicity and achievement. The following table collects the annual errors, in terms of percentage, obtained for the Standard & Poors index values in the period 2000-2007 for both approximations.

Year	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**
Linear	0,91%	0,81%	0,92%	0,64%	0,46%	0,45%	0,40%	0,73%
Spline	0,93%	0,84%	0,98%	0,70%	0,47%	0,47%	0,42%	0,80%

It is remarkable the fact that the errors corresponding to the linear interpolant are lower than those obtained for the cubic spline. At the same time, a polygonal is easier to implement than a smooth piecewise polynomial. Given that the results were very similar in all the cases, we opted for a linear approximant in order to perform the following computations.

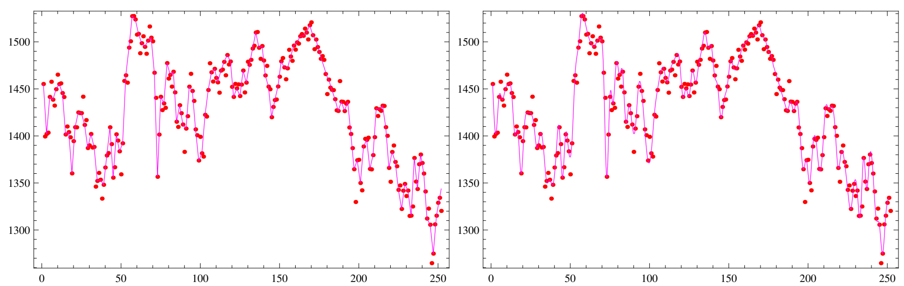

Fig. 2. Linear and spline interpolation of the Standard & Poor's data of the year 2000, along with the points not considered as nodes.

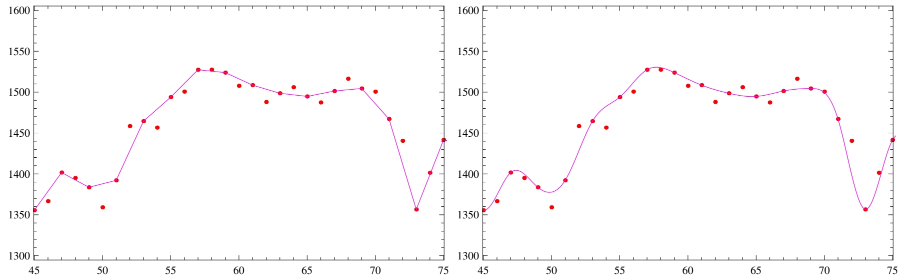

Fig. 3. Enlargement of Figure 2 in 30 trading days.

2.1.1. Numerical computation of the coefficients of an approximate Fourier curve

Since the Fourier methods are suitable for variables of stationary character, we subtracted previously the values on the regression line of the recording to the original data. Thus, the approximation curve consists of a linear part and a periodic component. We tested graphically that the computed curve fits the points well. Bearing in mind all these considerations, let M the number of terms considered in the Fourier sum. If $M = T$, only the annual cycles are collected, with $M = 12T$ we consider the monthly cycles and if $M = 52T$, up to the weekly cycles are gathered. We observed graphically and numerically the number of summands suitable to provide a fitting curve. One of these approximations is displayed in Figure 4. In this way, a set of harmonic amplitudes are obtained (c_m, d_m), corresponding to a sequence of angular frequencies $m{\cdot}\omega_0$, $(m = 0, 1, \ldots, M)$. The spectral powers are defined as $p_m = c_m^2 + d_m^2$.

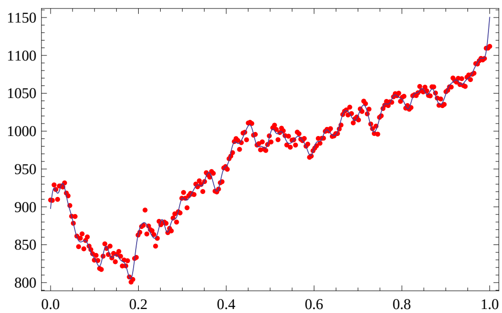

Fig. 4. Approximation curve for the values of the year 2003 of the S&P index, along with the data (in red).

2.2. Hjorth parameters of the approximation curve

In the seventies, the neurophysiologist B. Hjorth defined the descriptors Activity and Mobility in the international journal Electroencephalography and Clinical Neurophysiology ([2]). These quantifiers are based on the computation of the standard deviation of a signal and its derivatives and are useful to describe and assign numerical values to the different states of an experimental system or variable. The descriptor Activity gives a measurement of the amplitude of the signal and the Mobility provides an idea of the deviation of the derivative in reference to that of the amplitude.

The price of an index in a fixed period was considered as a function of time ($x(t)$), and as a function of the frequency as well, by mean of its Fourier Transform ($\widehat{x}(\omega)$). Multiplying $\widehat{x}(\omega)$ times its complex conjugate one obtains the power spectrum of the signal

$$S(\omega) = \widehat{x}(\omega)\widehat{x}^*(\omega).$$

The spectral moment of order n is defined as

$$m_n = \int_{-\infty}^{\infty} \omega^n S(\omega)\, d\omega, \qquad n = 0, 1, \ldots$$

The Hjorth parameters, hereafter $\mathcal{A}$ = Activity and $\mathcal{M}$ = Mobility, are expressed in terms of the spectral moments as

$$\mathcal{A} = m_0 = \int_{-\infty}^{\infty} S(\omega)\, d\omega$$

$$\mathcal{M} = \sqrt{\frac{m_2}{m_0}}.$$

These numbers can be computed in the time domain as well ([4], [3]). The Parseval relation implies that

$$m_0 = \frac{1}{T} \int_I |x(t)|^2 dt.$$

A similar relation for the derivative provides the following formula:

$$m_2 = \frac{1}{T} \int_I |\frac{dx}{dt}|^2 dt.$$

In our case, the original signal is not known except by its samples, and possibly it is not a smooth function. These facts motivate the computation of the parameters corresponding to a truncate Fourier sum, that is to say, in the frequency domain. One can prove that, beginning from the Fourier curve, the Hjorth descriptors adopt the expression, in terms of the numerical coefficients,

$$\mathcal{A} = m_0 = \overline{c_0}^2 + \frac{1}{2} \sum_{m=1}^{M} (\overline{c_m}^2 + \overline{d_m}^2)$$

$$\mathcal{M} = \left(\frac{m_2}{m_0}\right)^{1/2} = \frac{\omega_0 \left(\sum_{m=1}^{\infty} m^2 \left(\overline{c_m}^2 + \overline{d_m}^2\right)\right)^{1/2}}{\left(2\overline{c_0}^2 + \sum_{m=1}^{\infty} \left(\overline{c_m}^2 + \overline{d_m}^2\right)\right)^{1/2}}.$$

A difficulty in the use of the coefficients is their dependence of the magnitude of the original signal. Thus, the spectral moments depend on the amplitudes as well, and are not suitable to make accurate comparisons between different years and/or indices. To skip this inconvenience all the coefficients were normalized, dividing them by the maximum of the transformed price in the computed period.

The quotient m_2/m_0 turns out to be a weighted mean of the quadratic angular frequencies ($m\omega_0$), with weights equal to the spectral powers p_m. Extracting the square root, we obtain an averaged angular frequency of the signal in the period considered.

§3. Results

The results obtained for the different indices and years are collected in the following table:

	S&P		Nikkei		IBEX	
Year	**Activ.**	**Mobil.**	**Activ.**	**Mobil.**	**Activ.**	**Mobil.**
2000	0, 26	54, 25	0, 10	55, 33	0, 20	57, 57
2001	0, 37	41, 65	0, 17	45, 06	0, 25	53, 54
2002	0, 36	44, 13	0, 23	34, 48	0, 27	43, 70
2003	0, 10	49, 33	0, 26	46, 54	0, 14	52, 31
2004	0, 20	45, 63	0, 16	55, 39	0, 17	44, 88
2005	0, 27	51, 23	0, 15	26, 81	0, 14	45, 15
2006	0, 37	34, 69	0, 22	44, 50	0, 41	27, 21
2007	0, 28	51, 20	0, 24	44, 93	0, 31	52, 52
2008	0, 34	34, 11	0, 39	32, 94	0, 21	46, 70
2009	0, 07	61, 80	0, 21	46, 78	0, 07	60, 67
2010	0, 29	38, 30	0, 24	39, 42	0, 22	53, 21
2011	0, 31	60, 05	0, 17	66, 37	0, 30	64, 58
2012	0, 27	51, 10	0, 14	51, 49	0, 46	28, 35
2013	0, 16	66, 13	0, 08	51, 60	0, 27	35, 88

The spectral patterns of the indices are quite similar, excluding their behavior in mutual correlations. This fact points to their robustness (they are not due to pure chance or numerical artifacts). It also implies that there are "universal" spectral patterns, that is to say, the different international indices have a similar behavior in frequency. The correlation matrices for the parameters Activity and Mobility have been computed in the periods 2000 – 2006 and 2007 – 2013 in order to distinguish between pre-crisis period and the crisis period respectively.

	Activity 2000-2006		
	S&P	**Nikkei**	**IBEX**
S&P	1	−0, 09	0, 75
Nikkei	−0, 09	1	0, 27
IBEX	0, 75	0, 27	1

	Activity 2007-2013		
	S&P	**Nikkei**	**IBEX**
S&P	1	0, 45	0, 53
Nikkei	0, 45	1	−0, 38
IBEX	0, 53	−0, 38	1

	Mobility 2000-2006		
	S&P	**Nikkei**	**IBEX**
S&P	1	0, 094	0, 76
Nikkei	0, 094	1	0, 27
IBEX	0, 76	0, 27	1

	Mobility 2007-2013		
	S&P	**Nikkei**	**IBEX**
S&P	1	0, 75	0, 06
Nikkei	0, 75	1	0, 17
IBEX	0, 06	0, 17	1

In the second period the indices S & P and Nikkei increase significantly their correlation. However, specifically for the Mobility parameter, the IBEX suffers an alarming decorrelation in the second period, indicating that movements in the other indices are not corroborated by the Spanish selective.

It is worth noting that the Mobility parameter is between the fourth and eleventh harmonic for all indices considered, as shown in Figure 5.

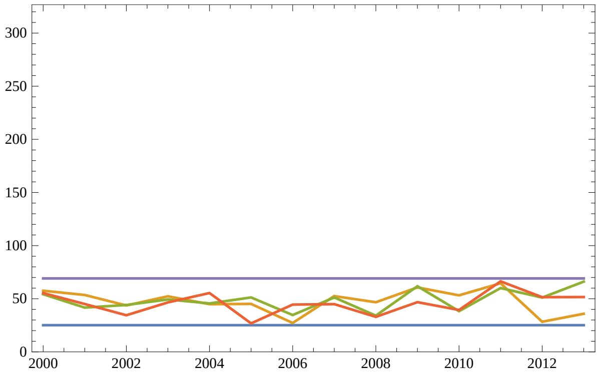

Fig. 5. Evolution of the Mobility of the indices through the period 2000-2013. The horizontal lines correspond to fourth and eleventh harmonic frequencies.

One of the objectives of the work was to find (if any) statistical differences in the parameters computed regarding the index considered. The samples were here the annual values of each selective. The parametric tests require some hypotheses on the variables like, for instance, normality, equality of variances, etc. In our case we cannot assume the normality of the distribution because is unknown. Commonly this condition is acceptable for large samples, but the size is small here, and for this reason we chose a non-parametric test, being Mann-Whitney a valid alternative. In all cases the corresponding p-value was greater than 0.05, except in the Activity for the index S&P and Nikkei. In this case, the null hypothesis can be rejected at the 95 % confidence level. For all the other cases, there is no statistically significant evidence of differences. In general, the p-values obtained in Activity are lower than for Mobility. This indicates that there is a higher probability in Mobility for the values to come from identical distributions, that is, in terms of the Mobility the indices present a stronger similitude.

§4. Conclusions

- The values of the Activity descriptor values are grouped at around $0,23$. The minor standard deviation is $0,08$ and corresponds to the Nikkei index. The greatest occur in the S&P and IBEX with a value of $0,10$. Note that the values have been normalized.
- Mobility of the considered indices rates among the fourth and eleventh harmonic with mean average of $47,41$.
- For both parameters the correlation between the parameters increases from the period 2000-2006 to 2007-2013 except for IBEX, which presents a high decorrelation with the rest.
- The statistical analysis of both descriptors reveals the similarity of the distribution of Hjorth parameters between different indices (except in the case of S&P and Nikkei for the Activity Index) at a confidence level of 95%. Furthermore, the similitude is greater in the Mobility since the p-values obtained are higher.
- The IBEX is decorrelated with other indices during crisis, both in the spectral domain (Mobility) and the temporal one (original signal).
- The linear interpolation is the best fitting method for the handled data regarding the standard procedures checked.

All these results suggest that the proposed quantifiers provide useful information for the implementation of new economical techniques for the prediction of large movements of the market, the definition of strategies for purchase-sale shares or the evaluation of financial assets.

Acknowledgements

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