

TURBULENCE MODELING CHALLENGES IN AIRSHIP CFD STUDIES

Kamal El Omari, Eric Schall,
Bruno Koobus and Alain Dervieux

Abstract. Turbulent separated flows around an airship-like geometry (a prolate spheroid 6:1) are investigated using three turbulence modelings based on statistical and Large Eddy Simulation (LES) approaches. The turbulence models used in the simulations are a standard high Reynolds $k-\varepsilon$ model, a Smagorinsky LES model and a variational multiscale LES one. The flow of interest is characterized by a relatively low Mach number ($Mach = 0.15$), an angle of attack set to 20° and a Reynolds number fixed to 4×10^4 . The three-dimensional compressible Navier-Stokes equations equipped with the previous turbulent models are discretized by a mixed finite element/finite volume method. The simulations show that the primary longitudinal vortex is predicted by the three models, but only the VMS-LES model predicts a secondary vortical flow structure that is observed in experimental studies.

Keywords: Turbulence, Reynolds Averaged Navier-Stokes, Large Eddy Simulation, Variational Multiscale, Prolate Spheroid, Finite Volume, Unstructured Grid

AMS classification:

§1. Introduction

In the context of an airship development program, we consider the numerical study of the flow around a flying generic airship. We are specially interested in flows involving rather large angle of attack around three-dimensional smooth bodies and in the phenomena induced by such flows, like separation-induced turbulent vortices. Problems involving flow separation in three-dimensional configurations is a very challenging topic in fluid dynamics research.

The investigated geometry is a prolate spheroid. It represents the closest shape to an airship geometry that can be found in the literature.

Two-equations statistical models are designed for providing steady mean flow fields. They generally rely on a Boussinesq turbulent viscosity. But this viscosity may be too large and may damp important steady and unsteady vortical flow structures. Non-equilibrium flows, as those arising close to leading edge at high angle of attack, are generally not accurately modeled. Our study starts with a set of results obtained with a standard two-equations model ($k-\varepsilon$) combined with the Reichardt wall law.

In contrast to statistical models, LES ones are designed to the numerical simulation of the smallest details with respect to the grid size. Two LES models are applied to the calculation

of this low-Reynolds flow: a Smagorinsky model which has been already applied with the same numerics in previous studies [1], and a variational multiscale model [2][3] which has the advantage to bring modeling only to the finest resolvable scales.

§2. Turbulence modelings

2.1. The $k - \varepsilon$ two-equations turbulence model

The compressible Reynolds averaged Navier-Stokes (RANS) equations are expressed as follows (in a two-dimensional form for sake of brevity, the three-dimensional expression is straightforward):

$$\frac{\partial W}{\partial t} + \nabla \cdot \mathcal{F}(W) = \nabla \cdot \mathcal{R}(W) \quad \text{where } W = (\rho, \rho v_1, \rho v_2, E)^T \tag{1}$$

where W is the fluid state vector, \mathcal{F} and \mathcal{R} are respectively the convective and diffusive fluxes given by

$$\mathcal{F}(W) = \begin{pmatrix} \mathcal{F}_1(W) \\ \mathcal{F}_2(W) \end{pmatrix}, \quad \mathcal{R}(W) = \begin{pmatrix} \mathcal{R}_1(W) \\ \mathcal{R}_2(W) \end{pmatrix},$$

in which

$$\mathcal{F}_1(W) = \begin{pmatrix} \rho v_1 \\ \rho v_1^2 + p \\ \rho v_1 v_2 \\ (E + p) v_1 \end{pmatrix}, \quad \mathcal{F}_2(W) = \begin{pmatrix} \rho v_2 \\ \rho v_1 v_2 \\ \rho v_2^2 + p \\ (E + p) v_2 \end{pmatrix},$$

$$\mathcal{R}_1(W) = \frac{\mu}{Re} \begin{pmatrix} 0 \\ \sigma_{11} \\ \sigma_{12} \\ v_1 \sigma_{11} + v_2 \sigma_{12} + \frac{\gamma}{Pr} \frac{\partial e}{\partial x_1} \end{pmatrix} + \frac{1}{R_t} \begin{pmatrix} 0 \\ \sigma_{11} - \frac{2}{3} \rho k R_t \\ \sigma_{12} \\ v_1 \sigma_{11} + v_2 \sigma_{12} - \frac{2}{3} \rho k v_1 R_t + \frac{\gamma}{Pr_t} \frac{\partial e}{\partial x_1} \end{pmatrix},$$

$$\mathcal{R}_2(W) = \frac{\mu}{Re} \begin{pmatrix} 0 \\ \sigma_{12} \\ \sigma_{22} \\ v_1 \sigma_{12} + v_2 \sigma_{22} + \frac{\gamma}{Pr} \frac{\partial e}{\partial x_2} \end{pmatrix} + \frac{1}{R_t} \begin{pmatrix} 0 \\ \sigma_{12} \\ \sigma_{22} - \frac{2}{3} \rho k R_t \\ v_1 \sigma_{12} + v_2 \sigma_{22} - \frac{2}{3} \rho k v_2 R_t + \frac{\gamma}{Pr_t} \frac{\partial e}{\partial x_2} \end{pmatrix},$$

with $E = \rho e + \frac{1}{2} \rho \|v\|^2 + \rho k$, $v = (v_1, v_2)^T$, $p = (\gamma - 1) \rho e$, $\sigma = \nabla v + \nabla^T v - \frac{2}{3} \nabla \cdot v I_d$, $\gamma = \frac{c_p}{c_v}$

The closure of system (1) is achieved here by a two-equation high Reynolds $k - \varepsilon$ model governed by the following equations

$$\frac{\partial W_t}{\partial t} + \nabla \cdot \mathcal{F}_t(W_t) = \nabla \cdot \mathcal{R}_t(W_t) + \Omega(W_t), \quad \text{with} \quad W_t = \begin{pmatrix} \rho k \\ \rho \varepsilon \end{pmatrix}.$$

where, for a two-dimensional flows, the fluxes $\mathcal{F}_t(W_t)$ and $\mathcal{R}_t(W_t)$ are given by

$$\mathcal{F}_t(W_t) = \begin{pmatrix} \mathcal{F}_{t1}(W_t) \\ \mathcal{F}_{t2}(W_t) \end{pmatrix}, \quad \mathcal{R}_t(W_t) = \begin{pmatrix} \mathcal{R}_{t1}(W_t) \\ \mathcal{R}_{t2}(W_t) \end{pmatrix}, \quad \text{in which}$$

$$\mathcal{F}_{t1}(W_t) = \begin{pmatrix} \rho v_1 k \\ \rho v_1 \varepsilon \end{pmatrix}, \quad \mathcal{F}_{t2}(W_t) = \begin{pmatrix} \rho v_2 k \\ \rho v_2 \varepsilon \end{pmatrix},$$

$$\mathcal{R}_{t1}(W_t) = \begin{pmatrix} \left(\frac{\mu}{Re} + \frac{1}{\sigma_k R_t} \right) \frac{\partial k}{\partial x_1} \\ \left(\frac{\mu}{Re} + \frac{1}{\sigma_\varepsilon R_t} \right) \frac{\partial \varepsilon}{\partial x_1} \end{pmatrix}, \quad \mathcal{R}_{t2}(W_t) = \begin{pmatrix} \left(\frac{\mu}{Re} + \frac{1}{\sigma_k R_t} \right) \frac{\partial k}{\partial x_2} \\ \left(\frac{\mu}{Re} + \frac{1}{\sigma_\varepsilon R_t} \right) \frac{\partial \varepsilon}{\partial x_2} \end{pmatrix}$$

and

$$\Omega(W_t) = \begin{pmatrix} -\rho \varepsilon + \mathcal{P} \\ -c_{\varepsilon 2} \frac{\rho \varepsilon^2}{k} + c_{\varepsilon 1} \frac{\varepsilon}{k} \mathcal{P} \end{pmatrix},$$

$$\mathcal{P} = -\frac{2}{3} \rho k \nabla \cdot v + \frac{1}{R_t} \left[-\frac{2}{3} (\nabla \cdot v)^2 + \sum_{i,j=1,2} \left[\left(\frac{\partial v_i}{\partial x_j} \right)^2 + \frac{\partial v_i}{\partial x_j} \frac{\partial v_j}{\partial x_i} \right] \right];$$

\mathcal{P} is the turbulent energy production. The turbulent Reynolds number R_t is obtained from the eddy viscosity μ_t :

$$\frac{1}{R_t} = \mu_t = c_\mu \frac{\rho k^2}{\varepsilon},$$

and the closure coefficients σ_k , σ_ε , $c_{\varepsilon 1}$, $c_{\varepsilon 2}$ and c_μ are set to their standard values:

$$\sigma_k = 1.0; \quad \sigma_\varepsilon = 1.3; \quad c_{\varepsilon 1} = 1.44; \quad c_{\varepsilon 2} = 1.92; \quad c_\mu = 0.09.$$

2.2. Large Eddy Simulation with Smagorinsky model

LES models are based on a spatial filtering of the Navier-stokes equations with respect to a filter width Δ . Only the large scales corresponding to the filtered flow variables are directly simulated, and modeling is introduced to take into account the effect of the unresolved subgrid scales on these large scales. In this study, the well known Smagorinsky model is used.

The filtered field of a quantity f in a given space domain \mathcal{D} is obtained by convolution with a filter function $G_\Delta(\mathbf{x})$ and is defined by

$$\bar{f}(\mathbf{x}, t) = \int_{\mathcal{D}} f(\mathbf{y}, t) G_\Delta(\mathbf{x} - \mathbf{y}) d\mathbf{y} \quad (2)$$

This quantity can then be written as $f = \bar{f} + f'$ where f' is the fluctuations of this function at scales smaller than the filter width Δ .

In addition, for compressible flows, a density-weighted filter (Favre filter) is introduced as $\tilde{f} = \frac{\bar{\rho}f}{\bar{\rho}}$. The filtered Navier-Stokes equations become after the application of Smagorinsky modeling:

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial \bar{\rho} \tilde{u}_i}{\partial x_j} = 0 \quad (3)$$

$$\frac{\partial (\bar{\rho} \tilde{u}_i)}{\partial t} + \frac{\partial (\bar{\rho} \tilde{u}_i \tilde{u}_j)}{\partial x_j} = -\frac{\partial \Pi}{\partial x_j} + \frac{\partial \left[(\mu + \mu_t) \left(2\tilde{S}_{ij} - \frac{2}{3}\tilde{S}_{ii}\delta_{ij} \right) \right]}{\partial x_j} \quad (4)$$

$$\frac{\partial (\bar{\rho} \tilde{e})}{\partial t} + \frac{\partial [(\bar{\rho} \tilde{e} + \Pi) \tilde{u}_j]}{\partial x_j} = \frac{\partial \left[\tilde{u}_j \mu \left(2\tilde{S}_{ij} - \frac{2}{3}\tilde{S}_{ii}\delta_{ij} \right) \right]}{\partial x_j} + \frac{\partial}{\partial x_j} \left[\left(\frac{C_p \mu_t}{Pr_t} + K \right) \frac{\partial \Theta}{\partial x_j} \right] \quad (5)$$

and the constitutive equations are

$$\Pi = \bar{\rho} R \Theta \quad , \quad \tilde{e} = C_v \Theta + \frac{1}{2} (\|\tilde{u}\|^2) \quad , \quad S_{ij} = \frac{1}{2} \left(\frac{\partial \mathbf{u}_i}{\partial \mathbf{x}_j} + \frac{\partial \mathbf{u}_j}{\partial \mathbf{x}_i} \right) \quad ,$$

$$\mu_t = \bar{\rho} (C_s \Delta)^2 \sqrt{2\tilde{S}_{ij}\tilde{S}_{ij}} \quad (6)$$

where μ_t is the eddy viscosity introduced by the Smagorinsky model (Eq. 6). The complete definition of μ_t needs the definition of the filter width Δ and the Smagorinsky constant C_s . This constant is set to 0.18 and the filter width associated to a tetrahedron T_j is defined by

$$\Delta_j = \sqrt[3]{Vol(T_j)} \quad (7)$$

The details of the derivation of the previous equations can be found in [4].

2.3. The Variational Multiscale modeling of turbulence

Unlike the classical LES approach, the variational multiscale LES (VMS-LES) model is not based on a spatial filtering of the Navier-Stokes equations but on a variational projection of these equations on a coarse-scales space and a fine-scales space [2]. The VMS-LES method separates the scales *a priori*, that is before the simulation is started. Furthermore, the VMS-LES method models the effects of the unresolved scales only in the equations governing the finest resolved scales and not on the equations governing the whole resolved scales as the LES method does.

Let Ω be the flow domain discretized by a tetrahedral mesh, from which a dual finite-volume mesh is derived [6]. Convective fluxes are treated here by a finite volume method in which a variable w is approximated first by a constant in each control volume as $w = \sum_i w_i \mathcal{X}_i$, where \mathcal{X}_i is the characteristic function corresponding to the control volume C_i associated with node i , and w_i denotes the constant value of w in this control volume. Next, this first-order approximation is transformed into a higher-order spatial discretization by a MUSCL approach [5]. The diffusive fluxes are treated by a finite-element method in which a flow variable w is approximated by a continuous piecewise linear function. This can be written as $w = \sum_i w_i \Phi_i$, where Φ_i is the P1 shape function associated with node i , and w_i denotes the value of w at this node.

The semi-discretization of the compressible Navier-Stokes equations by the mixed element volume method with mass lumping leads to the following system of equations

$$\left\{ \begin{array}{l} A(\mathcal{X}_i, \mathbf{W}) = \int_{\Omega} \frac{\partial \rho}{\partial t} \mathcal{X}_i d\Omega + \int_{\partial Sup \mathcal{X}_i} \rho \mathbf{u} \cdot \mathbf{n} \mathcal{X}_i d\Gamma = 0 \\ \mathbf{B}(\mathcal{X}_i, \Phi_i, \mathbf{W}) = \int_{\Omega} \frac{\partial \rho \mathbf{u}}{\partial t} \mathcal{X}_i d\Omega + \int_{\partial Sup \mathcal{X}_i} \rho \mathbf{u} \otimes \mathbf{u} \cdot \mathbf{n} \mathcal{X}_i d\Gamma \\ \quad + \int_{\partial Sup \mathcal{X}_i} P \mathbf{n} \mathcal{X}_i d\Gamma + \int_{\Omega} \sigma \nabla \Phi_i d\Omega = \mathbf{0} \\ C(\mathcal{X}_i, \Phi_i, \mathbf{W}) = \int_{\Omega} \frac{\partial E}{\partial t} \mathcal{X}_i d\Omega + \int_{\partial Sup \mathcal{X}_i} (E + P) \mathbf{u} \cdot \mathbf{n} \mathcal{X}_i d\Gamma \\ \quad + \int_{\Omega} \sigma \mathbf{u} \cdot \nabla \Phi_i d\Omega + \int_{\Omega} \lambda \nabla T \cdot \nabla \Phi_i d\Omega = 0 \end{array} \right. \quad (8)$$

where $\mathbf{W} = (\rho, \mathbf{u}, T)^T$ and $\partial Sup \mathcal{X}_i$ denotes the boundary of the support of \mathcal{X}_i , and \mathbf{n} is the outward normal to this support.

Let \mathcal{V}_{FV} and \mathcal{V}_{FE} denote respectively the space spanned by the characteristic functions $\{\mathcal{X}_k\}$, and that spanned by the P1 shape functions $\{\Phi_k\}$. In order to separate *a priori* the coarse and fine scales, these spaces are decomposed as follows

$$\mathcal{V}_{FV} = \overline{\mathcal{V}}_{FV} \oplus \mathcal{V}'_{FV} \quad , \quad \mathcal{V}_{FE} = \overline{\mathcal{V}}_{FE} \oplus \mathcal{V}'_{FE}. \quad (9)$$

Here the overline refers to the coarse scales, and the ' superscript to the fine scales. Hence, consistently with Eqs. (9), \mathbf{W} is decomposed into a coarse scale component $\overline{\mathbf{W}}$ and a fine scale part \mathbf{W}' as $\mathbf{W} = \overline{\mathbf{W}} + \mathbf{W}'$. This decomposition relies on coarse finite-volume cells which are built by agglomeration of a small number of fine dual control volumes.

>From this decomposition, the problem (8) is transformed into the two following subproblems

$$\left\{ \begin{array}{l} A(\overline{\mathcal{X}}_i, \overline{\mathbf{W}} + \mathbf{W}') = 0 \\ \mathbf{B}(\overline{\mathcal{X}}_i, \overline{\Phi}_i, \overline{\mathbf{W}} + \mathbf{W}') = \mathbf{0} \\ C(\overline{\mathcal{X}}_i, \overline{\Phi}_i, \overline{\mathbf{W}} + \mathbf{W}') = 0 \end{array} \right. \quad \left\{ \begin{array}{l} A(\mathcal{X}'_i, \overline{\mathbf{W}} + \mathbf{W}') = 0 \\ \mathbf{B}(\mathcal{X}'_i, \Phi'_i, \overline{\mathbf{W}} + \mathbf{W}') = \mathbf{0} \\ C(\mathcal{X}'_i, \Phi'_i, \overline{\mathbf{W}} + \mathbf{W}') = 0 \end{array} \right. \quad (10)$$

Without giving the details of the model derivation – that the reader can find in [3] –, we can say that this decomposition gives two sets of equations, one governing the coarse resolved scales and the other one the fine resolved scales. These sets of equations are coupled. Some terms that appear in the fine resolved scales equations require some modeling (unless all scales are resolved by means of DNS). This is achieved through a compressible generalized Smagorinsky eddy viscosity model involving only the fine resolved scales. Then, the final form of these

equations writes

$$\left\{ \begin{array}{l} \int_{\Omega} \frac{\partial \rho}{\partial t} \mathcal{X}_i d\Omega + \int_{\partial S_{sup} \mathcal{X}_i} \rho \mathbf{u} \cdot \mathbf{n} \mathcal{X}_i d\Gamma = 0 \\ \int_{\Omega} \frac{\partial \rho \mathbf{u}}{\partial t} \mathcal{X}_i d\Omega + \int_{\partial S_{sup} \mathcal{X}_i} \rho \mathbf{u} \otimes \mathbf{u} \mathbf{n} \mathcal{X}_i d\Gamma + \int_{\partial S_{sup} \mathcal{X}_i} P \mathbf{n} \mathcal{X}_i d\Gamma \\ + \int_{\Omega} \sigma \nabla \Phi_i d\Omega + \int_{\Omega} \tau' \nabla \Phi'_i d\Omega = \mathbf{0} \\ \int_{\Omega} \frac{\partial E}{\partial t} \mathcal{X}_i d\Omega + \int_{\partial S_{sup} \mathcal{X}_i} (E + P) \mathbf{u} \cdot \mathbf{n} \mathcal{X}_i d\Gamma + \int_{\Omega} \sigma \mathbf{u} \cdot \nabla \Phi_i d\Omega \\ + \int_{\Omega} \lambda \nabla T \cdot \nabla \Phi_i d\Omega + \int_{\Omega} \frac{C_p \mu'_t}{Pr_t} \nabla T' \cdot \nabla \Phi'_i d\Omega = 0 \end{array} \right. \quad (11)$$

where $\tau'_{ij} = \mu'_t (2S'_{ij} - \frac{2}{3} S'_{kk} \delta_{ij})$, $S'_{ij} = \frac{1}{2} (\frac{\partial \mathbf{u}'_i}{\partial \mathbf{x}'_j} + \frac{\partial \mathbf{u}'_j}{\partial \mathbf{x}'_i})$, $\mu'_t = \bar{\rho} (C'_s \Delta')^2 |S'|$, $|S'| = \sqrt{2S'_{ij} S'_{ij}}$, $C'_s = 0.1$ and Δ' denotes the local grid size. In this work, Δ' has the same definition as in Eq. (7).

2.4. Wall law

In all the simulations presented in this study, the flow domain is extended only up to a wall boundary located at a distance δ from the surface of the prolate. In the evaluation of the viscous fluxes, the wall shear stress is computed as $\tau_w = \rho u_f^2$ where the friction velocity u_f is determined from the non-linear Reichardt's law

$$U_{|\delta}^+ = 2.5 \log(1 + 0.41 \delta^+) + 7.8 \left(1 - e^{-\delta^+/11} - \frac{\delta^+}{11} e^{-0.33 \delta^+} \right) \quad (12)$$

in which

$$\delta^+ = \frac{u_f}{\nu} \delta \quad \text{and} \quad U_{|\delta}^+ = \frac{(v \cdot \mathbf{n}^\perp)_{|\delta}}{u_f}.$$

§3. Numerical issues

As stated before, the spatial discretization of the Navier-Stokes equation is achieved on unstructured tetrahedral meshes by a mixed element volume formulation (see [9, 8] for details).

- Convective terms are discretized by a finite volume method. Roe's scheme [7] is used together with a MUSCL approach [5, 6] to obtain a higher order of spatial accuracy (second-order was used in the scope of this paper).
- Diffusive fluxes are approximated by a Galerkin method using P_1 shape functions.
- Temporal scheme is implicit, second-order accurate.

The computer code is parallelized using a non-overlapping domain decomposition.

§4. Description of the test-case

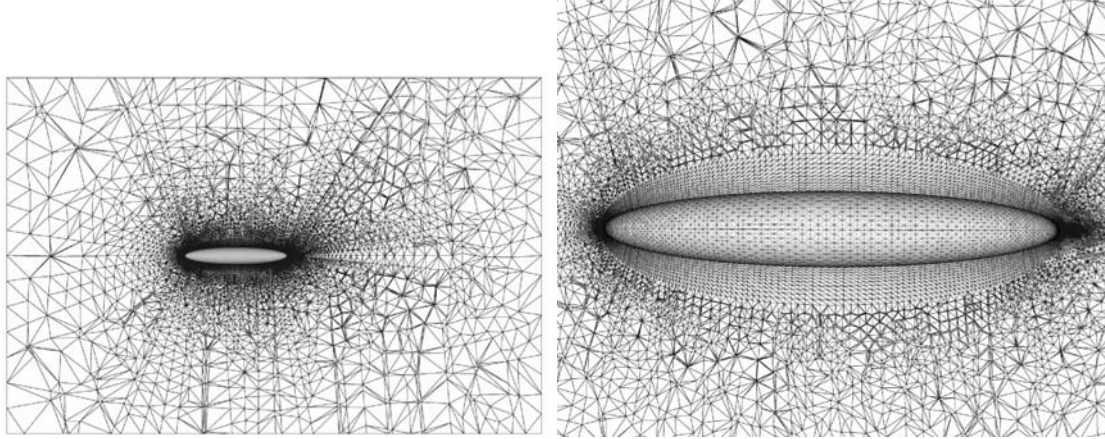


Figure 1: View of the mesh in the vertical symmetry plane (left) and zoom of the prolate (right)

We are interested in a flow around a 6:1 prolate spheroid of length $L = 1.37\text{ m}$, at 20° angle of attack. The flow has the following farfield characteristics

- Mach number $M_\infty = 0.15$;
- Reynolds number $Re = \frac{V_\infty L}{\nu} = 4 \times 10^4$;
- density $\rho_\infty = 1.1\text{ kg m}^{-3}$;
- pressure $p_\infty = 101300\text{ Pa}$;

The laminar-turbulent transition is fixed at $x/L = 0.2$.

The three-dimensional unstructured mesh used in the simulations is displayed in Fig. 1. It contains approximately 160000 nodes and 950000 tetrahedra. The computational domain dimensions are $7.2\text{ m} \times 4.8\text{ m} \times 4.8\text{ m}$. These dimensions were judged sufficient in previous numerical studies [10]. The mesh contains, close to the prolate, a pseudo-structured part for which the first elements height at the prolate surface is less than $7 \cdot 10^{-3}\text{ m}$. This value corresponds to a y^+ that lies between 4 and 23.

§5. Results

Three runs are performed corresponding to the turbulence models presented in this paper. Fig. 2 displays the isovalues of the streamwise velocity obtained with these models in cross-planes located on the rear part of the prolate. We can notice that the vortices predicted by LES and VMS-LES models are better captured, and that the VMS-LES calculation shows more flow details than the LES and RANS models. Indeed, these two last models tend to introduce a too large amount of turbulent viscosity which damps the turbulent structures.

Fig. 3 depicts the fluid velocity vectors and streamlines in the cross-plane located at $x = 1.2\text{ m}$. It confirms the previous observations, and we also notice that the VMS-LES model

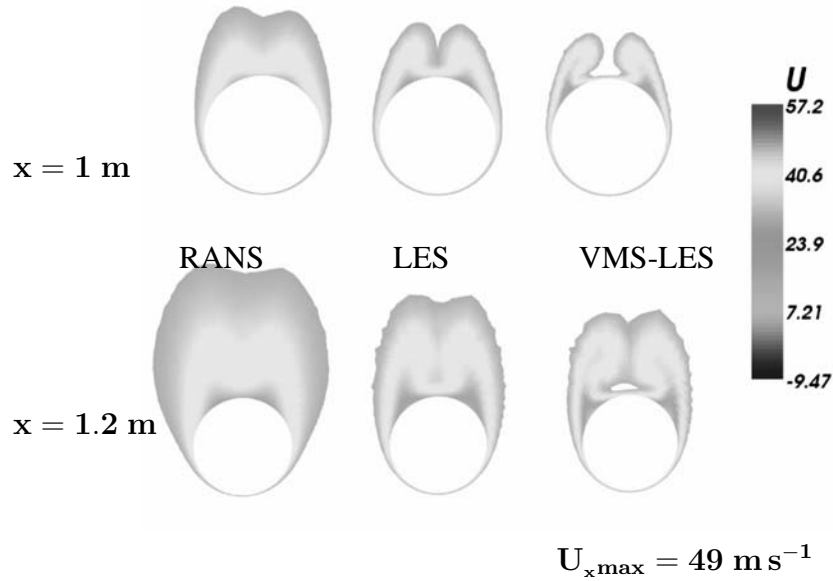


Figure 2: Streamwise velocity isovalues on cross-planes at $x = 1 \text{ m}$ and $x = 1.2 \text{ m}$

is able to capture a secondary vortex that is not predicted by the two others models. Such a vortical flow structure is effectively observed by experimental and numerical studies with higher Reynolds [11, 10].

We have seen that the three models exhibit perceptible differences concerning the longitudinal vortices, but the comparison of the behaviour of pressure at the prolate surface shows that this quantity remains almost the same, specially on the symmetry line as shown in Fig. 4.

§6. Conclusion

We have investigated the simulation of a low Mach compressible flow around a geometry close to an airship using three turbulent models based on statistical and LES approaches. With this geometry, the flow separation is due to adverse pressure gradients in the azimuthal direction and not to geometry singularities, in contrast to many vortex shedding simulations achieved with the VMS-LES model in previous works. The study shows that, for flows with high angle of attack and strong boundary layer separation, low viscosity models like the VMS-LES model, are more suitable for capturing the main vortical flow structures. With this low Reynolds case, the three models are not very CPU consuming and of about the same cost. But the use of LES based models may be prohibitive for high Reynolds number flow calculations. Then a good compromise would be a blend of VMS-LES and classical RANS models.

Acknowledgements

The authors are grateful to *Conseil Régional d'Aquitaine* for its financial support to the scientific airship project of Pau University and CINES for providing the computational resources.

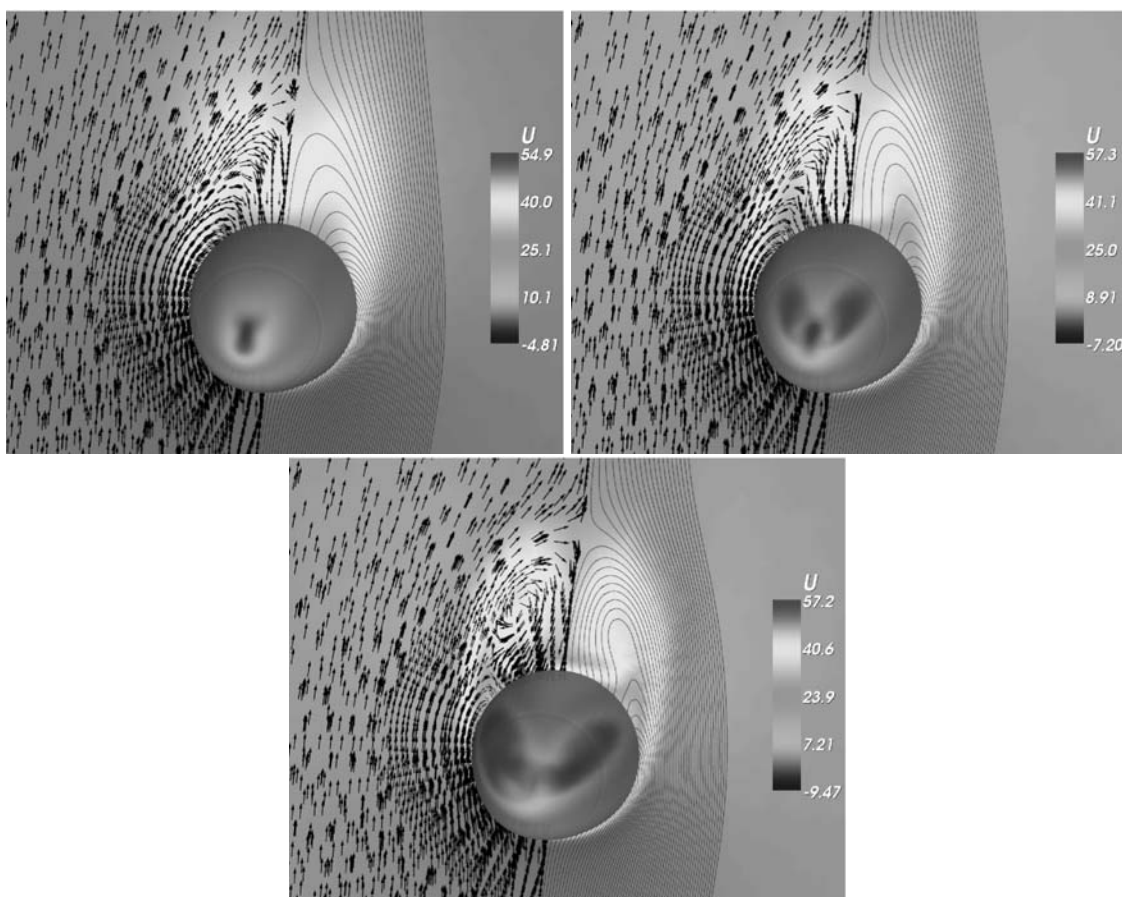


Figure 3: Velocity vectors and streamlines on cross-plane at $x = 1.2\text{ m}$: RANS (top left), LES (top right), VMS-LES(bottom)

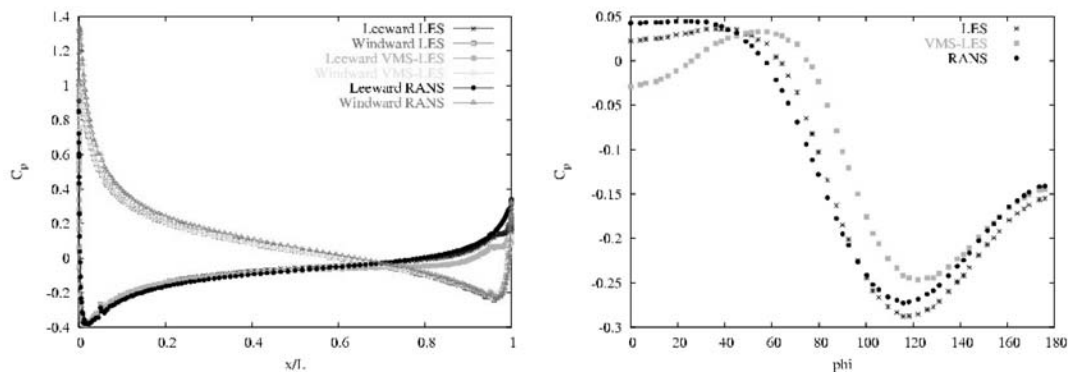


Figure 4: Pressure Coefficient C_p on the prolate surface at $z = 0$ (left) and at $x = 1.2\text{ m}$ (right)

References

- [1] S. CAMARRI, M.V. SALVETTI, B. KOOBUS AND A. DERVIEUX. Large-eddy simulation of a bluff-body flow on unstructured grids. *Int. J. Numer. Methods Fluids*, 40 (2002), 1431-1460.
- [2] T.J.R. HUGHES, L. MAZZEI AND K.E. JANSEN. Large eddy simulation and the variational multiscale method, *Comp. Vis. Sci.*, 3, 47, 2000.
- [3] B. KOOBUS AND C. FARHAT. A Variational Multiscale Method for the Large Eddy Simulation of Compressible Turbulent Flows on Unstructured Meshes - Application to Vortex Shedding. *Comput. Methods Appl. Mech. Eng.*, in press. (also published as *INRIA research report, N° 4722, February 2003*)
- [4] S. CAMARRI, M.V. SALVETTI, Towards the Large-Eddy simulation of complex engineering flows with unstructured grids. *INRIA research report, N° 3844, December 1999*.
- [5] B. VAN LEER, Towards the ultimate conservative difference scheme V: a second-order sequel to Godunov's method, *J. Comp. Phys.*, 32 (1979), 361-370.
- [6] A. DERVIEUX, Steady Euler Simulations Using Unstructured Meshes, *Von Karman Institute Lecture Series*, (1985).
- [7] P.L. ROE, Approximate Riemann solver, parameters vectors and difference schemes, *J. Comp. Phys.*, 43 (1981), 357-371.
- [8] B. KOOBUS, C. FARHAT. AND H. TRAN, Computation of unsteady viscous flows around moving bodies using the $k-\varepsilon$ turbulent model on unstructured dynamic grids. *Comput. Methods Appl. Mech. Engrg.*, 190 (2000), 1441-1466.
- [9] S. CAMARRI, M.V. SALVETTI, A. DERVIEUX. AND B. KOOBUS A low diffusion MUSCL scheme for LES on unstructured grids. *INRIA research report, N° 4512, July 2002*.
- [10] G.S. CONSTANTINESCU, H. PASINATO, Y.Q. WANG, J.R. FORSYTHE AND K.D. SQUIRES. Numerical Investigation on Flow Past a Prolate Spheroid. *J. Fluids Eng.*, 124 (2002), 904-910.
- [11] CHESNAKAS C. J AND SIMPSON R. L A Detailed Investigation of the 3-D Separation about a 6:1 Prolate Spheroid at Angle of Attack. *AIAA Journal*, 35, 6 (1997), 990-999.

K. El Omari and E. Schall

IUT GTE, UPPA

1 Av. de l'Université

64000 Pau

kamal.elomari@univ-pau.fr and eric.schall@univ-pau.fr

B. Koobus

Université de Montpellier II

Place Eugène Bataillon, dépt. Math.

CC 051, F-34095 Montpellier Cedex 5

koobus@darboux.math.univ-

montp2.fr

A. Dervieux

INRIA, 2003 Route des Lucioles

F-06902 Sophia-Antipolis Cedex

alain.dervieux@inria.fr