WAVE SCATTERING BY A PERIODIC ARRAY OF IN-PLANE CRACKS AT THE INTERFACE BETWEEN DISSIMILAR MEDIA

Nathalie Favretto-Cristini

Abstract. We investigate analytically the behaviour of time-harmonic elastic waves in the neighbourhood of a plane interface between dissimilar elastic media, where a periodic array of in-plane cracks exists. Waves incident on such a boundary excite a lot of propagating and evanescent scattered waves, and the incident energy is subjected to a complicated process of redistribution. The aim of the work is to quantify the amount of incident wave energy partitioned among the propagating scattered waves. The scattered fields are expressed in terms of Fourier series with coefficients depending on Legendre functions. Energies associated with the scattered waves are evaluated as a function of the incidence angle, and results are presented for three different distributions of the cracks at the interface. We show that the amount of the reflected wave energy and the amount of the diffracted wave energy increase with increasing percentage of cracks at the interface. We also suggest that, contrary to the case of an interface between identical media, the energy conservation law cannot be applied in the work presented here since phenomena associated with interface wave scattering are not taken into account.

§1. Introduction

Intensive studies have addressed the problem of elastic wave scattering by a periodic array of cracks because of its conceptual and practical importance in non destructive testing of materials, for example. Angel and Achenbach [1] have presented an exact analysis of the reflection of elastic waves by a planar array of periodically spaced cracks of equal lengths. By the use of Fourier series techniques, the mixed-boundary value problem for a typical strip is reduced to a singular integral equation of the 1st kind for the dislocation density across the crack faces. The equation is then solved numerically. The exact results are, however, rather complicated. The applicability of an approximate solution to the exact problem was investigated in numerous papers (see ref. [2] and the excellent review given there), in which the array of cracks are generally replaced by a layer of massless springs. The interface stiffnesses are chosen so that the spring layer produces the same static displacements as the array of cracks, when the elastic medium is subjected to distant uniform tension. Angel and Achenbach [3] have considered this quasi-static model as a low-frequency limit of the exact solution. In this case, only one specular reflected wave has to be taken into account at some distance from the plane of cracks. However, as the ratio of incident wavelength-to-array period decreases, the quasi-static model becomes invalid, since more and more propagating reflected waves which travel into off-specular directions are generated by the secondary sources (i.e., cracks) at the interface.

Recently, Danicki has studied this problem from a very particular point of view [4, 5]. To solve the problem of wave scattering by a periodic array of cracks, the BIS method [6] has been applied. The method exploits some properties of Fourier series with coefficients expressed by Legendre functions. Three features make the use of Legendre functions particularly well-suited for modeling elastic wave fields by a periodic array of cracks. First, the series are periodic, as required by Floquet's theorem. Moreover, the identities concerning Legendre functions satisfy implicitly the mixed-boundary conditions. Finally, they also exhibit square-root singularity, in correspondence with the singularity of the wavefields at the crack edges. The solution to the wave-scattering problem can thus be obtained efficiently in an analytical way.

In the present paper, using the fundamentals of the BIS method, we investigate the complicated process of redistribution of the incident energy (associated either with a compressional wave, called a P-wave, or with a transverse wave, called a S-wave), as a function of the incidence angle, among the propagating scattered waves in the vicinity of a solid/solid interface with air (or gas)-filled cracks. For the sake of brevity, the influence of the characteristics of the crack array and the influence of the properties of the incident wave are shown. Numerical results, presented here for applications of geophysical interest [7], viz. the interface between chalk and granite, concern only the incident P-wave.

§2. Description of the configuration and formulation of the problem

We investigate the scattering of time-harmonic elastic waves by a periodic array of cracks at the boundary between two dissimilar media. The media are assumed to be homogeneous, isotropic, and perfectly elastic half-spaces, with mass density ρ, P-wave velocity C_P and S-wave velocity C_S. We refer to the upper medium ($x_2 < 0$) as solid B and to the lower medium ($x_2 > 0$) as solid A. The cracks lie in the plane $x_2 = 0$ and extend to infinity in the direction perpendicular to the ($\mathbf{x_1} - \mathbf{x_2}$) plane. The excitation being assumed to be independent of the $\mathbf{x_3}$-direction, the problem is a two-dimensional one. The period of the crack array is Λ; $K = \frac{2\pi}{\Lambda}$ denotes the spatial wavenumber. The regions of perfect bonding between the elastic half-spaces are $2w$ wide.

We consider a time-harmonic plane wave, characterized by the wavelength λ_{inc}, and angular frequency ω, that propagates in the solid B and hits the interface under the incidence angle α_{inc}, with respect to the normal to the interface. The associated incident particle displacement vector $\mathbf{U_{inc}}$ in the solid B can be written as:

$$\mathbf{U_{inc}}(x_1, x_2; t) = \mathbf{u_{inc}}(x_1, x_2) \exp(-jq_{inc}x_2) \exp(-jp_{inc}x_1) \exp(j\omega t) \tag{1}$$

where $\mathbf{u_{inc}} = (\sin\alpha_{inc}, \cos\alpha_{inc})$ is the unit propagation vector of the incident P-wave, or $\mathbf{u_{inc}} = (\cos\alpha_{inc}, -\sin\alpha_{inc})$ is the unit propagation vector of the incident S-wave, and $\mathbf{k_{inc}} = (p_{inc}, q_{inc}) = \mathbf{k^B_{P,S}}$ the incident P- or S-wavevector. The amplitude of the incident displacement vector is assumed to be unity. At the interface, the incident displacement vector $\mathbf{U_{inc}}$ and the traction-force vector $\mathbf{T_{inc}}$ are related by:

$$\mathbf{U_{inc}} = \overline{\mathbf{G'_{inc}}}\,\mathbf{T_{inc}} \tag{2}$$

where the matrix $\overline{\mathbf{G}'_{\mathbf{inc}}}$, given below, is defined from the known equations of motion [8], in which absence of body forces is assumed:

$$T_{ij,j} + \rho\omega^2\, U_i = 0 \tag{3}$$

$_{,j}$ denotes partial differentiation with respect to x_j.

After hitting the cracked interface, the incident wave excites a lot of diffracted waves that propagate in different directions. At the interface, the full wavefield (particle displacement vector $\mathbf{U^{A,B}_{tot}}$and traction-force vector $\mathbf{T^{A,B}_{tot}}$) in each medium, governed by equations of motion 3, is expressed in the form:

$$\left\{\begin{array}{l}\mathbf{U^A_{tot}} = \mathbf{U^A_{diff}}\\ \mathbf{T^A_{tot}} = \mathbf{T^A_{diff}}\end{array}\right. \qquad \text{and} \qquad \left\{\begin{array}{l}\mathbf{U^B_{tot}} = \mathbf{U_{inc}} + \mathbf{U^B_{diff}}\\ \mathbf{T^B_{tot}} = \mathbf{T_{inc}} + \mathbf{T^B_{diff}}\end{array}\right.$$

$\mathbf{U^{A,B}_{diff}}$and $\mathbf{T^{A,B}_{diff}}$ characterize the diffracted fields in the solids A and B. At the interface, they are formulated as an infinite series of Bloch waves, as required by Floquet's theorem [8]:

$$\left[\begin{array}{c}\mathbf{U^{A,B}_{diff}}\\ \mathbf{T^{A,B}_{diff}}\end{array}\right] = \sum_{n=-\infty}^{+\infty}\left[\begin{array}{c}\mathbf{U^{A,B}_{n}}\\ \mathbf{T^{A,B}_{n}}\end{array}\right]\exp\left(-jp_n x_1\right)\,\exp\left(j\omega t\right) \tag{4}$$

where $p_n = p_{inc} + nK \;\; (0 < p_{inc} < K)$ is the Bloch wavenumber and n the diffraction order. The zeroth-diffracted waves correspond to classical reflections and transmissions, while the n^{th}-diffracted waves (with n different from zero) correspond to off-specular reflections and transmissions induced by the secondary sources at the interface (i.e., by the periodic array of cracks). p_{inc} is the horizontal wavenumber common to all waves of diffraction order zero. The scattered far-field consists of a superposition of a finite number of propagating P- and S-wave components, with the general form $\exp\left(-jp_n x_1\right)\,\exp\left(\pm j\left(q^{A,B}_{P,S}\right)_n x_2\right)\,\exp\left(j\omega t\right)$. In order to satisfy the radiation conditions by the wavefield in the half-spaces, the values of $\left(q^{A,B}_{P,S}\right)_n$ are chosen following the rule $\left(q^{A,B}_{P,S}\right)_n = \left[\left(k^{A,B}_{P,S}\right)^2 - p_n^2\right]^{\frac{1}{2}} = -j\left[p_n^2 - \left(k^{A,B}_{P,S}\right)^2\right]^{\frac{1}{2}}$, with P- and S-wavenumbers $k^{A,B}_{P,S}$.

We consider the coefficients $\mathbf{U^{A,B}_{n}}$and $\mathbf{T^{A,B}_{n}}$ of eq.4 as the fundamental quantities to be determined. At the interface, these coefficients are related by:

$$\mathbf{U^{A,B}_{n}} = \overline{\mathbf{G^{A,B}_{n}}}\,\mathbf{T^{A,B}_{n}} \tag{5}$$

where the matrices $\overline{\mathbf{G^{A,B}_{n}}}$ are defined from the equations of motion 3:

$$\overline{\mathbf{G^A_n}} = \frac{j}{\mu_A D_A}\left[\begin{array}{cc}\left(k^A_S\right)^2\left(q^A_S\right)_n & p_n\left(\left(k^A_S\right)^2 - 2p_n^2 - 2\left(q^A_P\right)_n\left(q^A_S\right)_n\right)\\ -p_n\left(\left(k^A_S\right)^2 - 2p_n^2 - 2\left(q^A_P\right)_n\left(q^A_S\right)_n\right) & \left(k^A_S\right)^2\left(q^A_P\right)_n\end{array}\right]$$

$$\overline{\mathbf{G^B_n}} = -^t\overline{\mathbf{G}'_{\mathbf{n}}}$$

$\mu_A = \rho_A\left(C^A_S\right)^2$ is the Lame coefficient of solid A, and $D_A = \left[\left(k^A_S\right)^2 - 2p_n^2\right]^2 + 4p_n^2\left(q^A_P\right)_n\left(q^A_S\right)_n$ is the known characteristic equation of the Rayleigh wave propagating at the free surface of solid A [8]. ${}^t\overline{\mathbf{G'_n}}$ is transposed to $\overline{\mathbf{G'_n}}$. $\overline{\mathbf{G'_n}}$, and also $\overline{\mathbf{G'_{inc}}}$, is similar to $\overline{\mathbf{G_n^A}}$ calculated using the properties of solid B.

In this paper, we investigate wave scattering by a periodic array of thin air-filled cracks at the boundary between two welded elastic half-spaces. The resulting mixed-boundary conditions, satisfied by the full-wavefields at $x_2 = 0$, are thus:

$$\left\{ \begin{array}{c} \mathbf{T_{tot}^A} = \mathbf{T_{tot}^B} \\ \Delta\mathbf{U} = \mathbf{U_{tot}^A} - \mathbf{U_{tot}^B} = \mathbf{0} \end{array} \right. \text{ between cracks,} \tag{6}$$

$$\mathbf{T_{tot}^A} = \mathbf{T_{tot}^B} = \mathbf{0} \text{ on cracks,} \tag{7}$$

where the function $\Delta\mathbf{U}$ denotes the particle displacement discontinuity at the interface. In order to simplify these expressions, we introduce the auxiliary function $\mathbf{V}\,(x_1)$ [4]:

$$\mathbf{V}\,(x_1) = \frac{\partial}{\partial x_1}\,(\Delta\mathbf{U}\,(x_1)) = \sum_{n=-\infty}^{+\infty} \mathbf{V_n}\,\exp\left(-jp_n x_1\right)\,\exp\left(j\omega t\right)$$

where

$$\begin{array}{l} \mathbf{V_n} = \overline{\mathbf{g_n}}\,\mathbf{T_n^A} - \overline{\mathbf{g_{inc}}}\,\mathbf{T_{inc}}\delta_{n0} \\ \left\{ \begin{array}{c} \overline{\mathbf{g_n}} = -jp_n\,\left[\overline{\mathbf{G_n^A}} + {}^t\,\overline{\mathbf{G'_n}}\right] \\ \overline{\mathbf{g_{inc}}} = -jp_{inc}\,\left[\overline{\mathbf{G'_{inc}}} + {}^t\,\overline{\mathbf{G'_{inc}}}\right] \end{array} \right. \end{array} \tag{8}$$

It has to be noted that the function $\mathbf{V}\,(x_1)$ determines $\Delta\mathbf{U}\,(x_1)$ within a constant. The boundary conditions 6 and 7 can then be reformulated, just for one period of the crack array, as:

$$\text{between cracks, i.e.}\ \forall x_1\ \mid |x_1| < w,\ \left\{ \begin{array}{c} \mathbf{V}\,(x_1) = \mathbf{0} \\ \Delta\mathbf{U}\,(x_1 = 0) = \mathbf{0} \end{array} \right. \tag{9}$$

$$\text{on cracks, i.e.}\ \forall x_1\ \mid \left\{ \begin{array}{c} -\Lambda + w < x_1 < -w \\ w < x_1 < \Lambda - w \end{array} \right. ,\ \mathbf{T_{tot}^A} = \mathbf{0} \tag{10}$$

§3. Reformulation of the problem and solution

An efficient way of solving such a problem can be found in [4]. Nevertheless, as the objectives of our work are completely different from those described in the paper, we need to derive equations for the case of a periodic array of cracks at the interface between two different elastic media. We then adapted the original work to our configuration. Refering the reader for details to the original paper [4], we write down only the most relevant equations that are necessary for the comprehension of our work.

Refering to some interesting properties of the Legendre functions P_ν of the 1^{st}-kind and degree ν involved in Fourier series [6, 9], we note that the periodicity of the problem, the exhibition of the square-root singularities at the crack edges, and the boundary conditions 10 and 9 are automatically satisfied if the functions $\mathbf{V_n}$ and $\mathbf{T_n^A}$ are searched in the general form:

$$\mathbf{T_n^A} = \sum_{m=M_1}^{M_2} \mathbf{t_m}\, P_{n-m}\left(\cos\Delta\right) \tag{11}$$

$$\mathbf{V_n} = \sum_{m=M_1}^{M_2} S_{n-m}\,\mathbf{v_m}\, P_{n-m}\left(\cos\Delta\right) \tag{12}$$

where $S_\nu = \left| \begin{array}{ll} 1 & (\nu \geq 0) \\ -1 & (\nu < 0) \end{array} \right.$ $(\nu \in \mathbb{Z})$. The variable $\Delta = K\,w = \pi\,\frac{2w}{\Lambda}$ describes the relative width of perfect contact between cracks. The summation limits M_1 and M_2 over m are supposed to be large but finite.

Substitution of $\mathbf{T_n^A}$ and $\mathbf{V_n}$ in 8 by their representation 11 and 12 leads to:

$$\forall n \in \left]-\infty,\ +\infty\right[,\quad \sum_{m=M_1}^{M_2} S_{n-m}\,\mathbf{v_m} P_{n-m}\left(\cos\Delta\right) = \overline{g_n} \sum_{m=M_1}^{M_2} \mathbf{t_m} P_{n-m}\left(\cos\Delta\right) - \overline{\mathbf{g_{inc}}}\,\mathbf{T_{inc}}\delta_{n0} \tag{13}$$

In order to reduce the number of unknowns in the problem, a relation between $\mathbf{v_m}$ and $\mathbf{t_m}$ is established from 13 following the same rule as that explained in [4]:

$$\left\{ \begin{array}{ll} \forall n\, \geqslant 0,\ \forall m\, \in\, [M_1,\, M_2] & \mathbf{v_m} = \overline{\mathbf{g}_\infty}\,\mathbf{t_m} \\ \forall n\, < 0,\ \forall m\, \in\, [M_1,\, M_2] & \mathbf{v_m} = {}^t\overline{\mathbf{g}_\infty}\,\mathbf{t_m} \end{array} \right. \tag{14}$$

where $\overline{\mathbf{g}_\infty}$ and ${}^t\overline{\mathbf{g}_\infty}$ are the asymptotic limits of the matrix $\overline{\mathbf{g_n}}$, such that:

$$\left\{ \begin{array}{lll} \forall n \in \left]-\infty,\ N_1\right[\ ,\ p_n < -p_\infty, & \lim\limits_{n\to-\infty} \overline{\mathbf{g_n}}\left(p_n\right) = S_{p_n}{}^t\overline{\mathbf{g}_\infty} \\ \forall n \in \left]N_2,\ +\infty\right[\ ,\ p_n > p_\infty, & \lim\limits_{n\to+\infty} \overline{\mathbf{g_n}}\left(p_n\right) = S_{p_n}\overline{\mathbf{g}_\infty} \end{array} \right. \tag{15}$$

with

$$\overline{\mathbf{g}_\infty} = \frac{1}{2\omega^2} \left[\begin{array}{cc} j\left(X_A\, \left(k_S^A\right)^2 + X_B\, \left(k_S^B\right)^2\right) & \left(X_A\, \left(k_P^A\right)^2 -\, X_B\, \left(k_P^B\right)^2\right) \\ -\left(X_A\, \left(k_P^A\right)^2 -\, X_B\, \left(k_P^B\right)^2\right) & j\left(X_A\, \left(k_S^A\right)^2 + X_B\, \left(k_S^B\right)^2\right) \end{array} \right]$$

and $X_{A,B} = \dfrac{\left(k_S^{A,B}\right)^2}{\rho_{A,B}\,\left(\left(k_S^{A,B}\right)^2 - \left(k_P^{A,B}\right)^2\right)}$. $N_1 < 0$, $N_2 > 0$, and p_∞ are sufficiently large.

For an interface between two elastic media with different properties, we can note that the asymptotic limits of $\overline{\mathbf{g_n}}$ are quite different from the asymptotic limit determined for an interface between identical media and reported in [4]. From the approximation 14, it follows that 13 can be rewritten only in terms of the unknowns $\mathbf{t_m}$ ($\mathbf{T_{inc}}$ being known from 2):

$$\begin{array}{l} \forall n\, \in\, \left[N_1,\, 0\right[,\ \sum\limits_{m=M_1}^{M_2} \left(S_{n-m}\,{}^t\overline{\mathbf{g}_\infty} - \overline{\mathbf{g_n}}\right)\,\mathbf{t_m}\, P_{n-m}\left(\cos\Delta\right) = \mathbf{0} \\ \forall n\, \in\, \left[0,\, N_2\right],\ \sum\limits_{m=M_1}^{M_2} \left(S_{n-m}\,\overline{\mathbf{g}_\infty} - \overline{\mathbf{g_n}}\right)\,\mathbf{t_m}\, P_{n-m}\left(\cos\Delta\right) = -\overline{\mathbf{g_{inc}}}\,\mathbf{T_{inc}}\delta_{n0} \end{array} \tag{16}$$

Eqs.16 express implicitly the boundary condition 10 and the first relation of the boundary condition 9. After some algebric manipulations, the second relation of the boundary condition 9 can be written for $r \neq 0$ and n fixed arbitrarily (here, n=0), with help of Dougall's expansion [9] and eq.14, in the form:

$$\sum_{m=M_1}^{M_2} (-1)^m \; \overline{\mathbf{g}_\infty} \, \mathbf{t_m} \, P_{-m-\frac{p_{inc}}{K}} \left(-\cos\Delta\right) = \mathbf{0} \tag{17}$$

Further analysis shows that the summation limits M_1 and M_2 over m can be defined as (see discussion in [4] and in the references given there): $M_1 = N_1 \leq 0 \text{ and } M_2 = N_2 + 1 > 0$.

Solving the scattering problem and recovering the diffracted wavefields in media then require to find a nontrivial solution to the linear system of $[N_2 - N_1 + 2]$ equations (i.e. eqs.16 and 17) on $[N_2 - N_1 + 2]$ unknowns (i.e. $\mathbf{t_m}$). The summation limits N_1 and N_2 over n are dependent on the degree of accuracy that is judiciously chosen for the approximations 15 of the matrix $\overline{\mathbf{g_n}}$, and also on the physical behaviour of the cracked interface. In particular, all the propagating diffraction orders must be taken into account.

§4. Evaluation of the scattered energy

Since we are mainly interested in the redistribution of the incident wave energy among the propagating diffracted waves, we introduce the time-averaged energy flux $\mathbf{\Pi}$ in the $\mathbf{x_2}$-direction for each diffraction order n and for the incident wave [8]:

$$\begin{aligned} \left(\mathbf{\Pi_n^{A,B}}\right)_{x_2} &= -\tfrac{1}{2}\,\Re\left[\left(\mathbf{T_n^{A,B}}\right)_{21}\,\left(j\omega\,\left(\mathbf{U_n^{A,B}}\right)_1\right)^* + \left(\mathbf{T_n^{A,B}}\right)_{22}\,\left(j\omega\,\left(\mathbf{U_n^{A,B}}\right)_2\right)^*\right] \\ \left(\mathbf{\Pi_{P,Sinc}}\right)_{x_2} &= -\tfrac{1}{2}\,\Re\left[\left(\mathbf{T_{P,Sinc}}\right)_{21}\,\left(j\omega\, U_1^{P,Sinc}\right)^* + \left(\mathbf{T_{P,Sinc}}\right)_{22}\,\left(j\omega\, U_2^{P,Sinc}\right)^*\right] \end{aligned} \tag{18}$$

where the asterisk denotes complex-conjugate quantities and $\Re(x)$ the real part of x. The displacement vector $\mathbf{U_n^{A,B}}$ and the traction force vector $\mathbf{T_n^{A,B}}$ are defined from the solution to the system of equations described in the previous section and from eq.5, while $\mathbf{U_{P,Sinc}}$ and $\mathbf{T_{P,Sinc}}$ are expressed from eqs 1 and 2. On the strength of eqs 18 and 5, the energy associated with the propagating nth-diffracted P- and S-waves can be defined in the vicinity of the cracked interface by:

$$\left(\mathbf{\Pi_{P,S}^{A,B}}\right)_{x_2}^{n} = \frac{1}{2}\,\rho_{A,B}\,\omega^3\,\Re\left[\left|\left(F_{P,S}^{A,B}\right)_n\right|^2\,\left(q_{P,S}^{A,B}\right)_n\right]$$

where the variables $\left(F_{P,S}^{A,B}\right)_n$ are expressed as a function of the traction force vector components [7]:

$$\begin{bmatrix} \left(F_P^A\right)_n \\ \left(F_S^A\right)_n \end{bmatrix} = \frac{j}{\mu_A\, D_n^A} \begin{bmatrix} 2p_n\,\left(q_S^A\right)_n & \left(k_S^A\right)^2 - 2p_n^2 \\ -\left[\left(k_S^A\right)^2 - 2p_n^2\right] & 2p_n\,\left(q_P^A\right)_n \end{bmatrix} \begin{bmatrix} \left(T_n^A\right)_{21} \\ \left(T_n^A\right)_{22} \end{bmatrix}$$

$$\begin{bmatrix} \left(F_P^B\right)_n \\ \left(F_S^B\right)_n \end{bmatrix} = \frac{j}{\mu_B\, D_n^B} \begin{bmatrix} 2p_n\,\left(q_S^B\right)_n & -\left[\left(k_S^B\right)^2 - 2p_n^2\right] \\ \left(k_S^B\right)^2 - 2p_n^2 & 2p_n\,\left(q_P^B\right)_n \end{bmatrix} \begin{bmatrix} \left(T_n^B\right)_{21} \\ \left(T_n^B\right)_{22} \end{bmatrix}$$

Since both media are lossless, we suppose that the energy flux interferences between the different diffraction orders can be neglected [10], and that the diffracted and incident fields then satisfy the following energy conservation law:

$$1 + \frac{\sum_n \left(\mathbf{\Pi_n^B}\right)_{x_2}}{\left(\mathbf{\Pi_{P,Sinc}}\right)_{x_2}} - \frac{\sum_n \left(\mathbf{\Pi_n^A}\right)_{x_2}}{\left(\mathbf{\Pi_{P,Sinc}}\right)_{x_2}} = 0 \tag{19}$$

where all summations are to be carried out over the collection of the pertaining propagating orders.

Figure 1 illustrates the changes in the ratio $\left|\frac{\left(\mathbf{\Pi_{P,S}^{A,B}}\right)_{x_2}^n}{\left(\mathbf{\Pi_{P,Sinc}}\right)_{x_2}}\right|$, associated with the different diffraction orders in the chalk and granite materials, as a function of the incidence angle α_{inc} associated only with the incident P-wave. The properties of the materials are: $\rho_B = 1180\,kg/m^3$, $C_{PB} = 2670\,m/s$, and $C_{SB} = 1120\,m/s$ for chalk, $\rho_A = 2700\,kg/m^3$, $C_{PA} = 6440\,m/s$, and $C_{SA} = 3170\,m/s$ for granite. The computations were carried out for three different crack distributions at the interface: 1% of cracks (i.e. $2w/\Lambda = 0.99$) which represents the case of a quasi-perfectly welded interface; 50% of cracks (i.e. $2w/\Lambda = 0.5$); 75% of cracks (i.e. $2w/\Lambda = 0.25$). The incident P-wavelength λ_{inc} in chalk was chosen to be greater than the spatial wavelength Λ of the crack distribution at the interface ($\Lambda = \frac{1}{3}\,\lambda_{inc}$). In this case, only one diffracted S-wave of order -1 is propagative in chalk, and only for the incidence angles α_{inc} greater than 38. The critical incident angle, at which the P-wave (respectively, S-wave) transmitted in granite becomes an inhomogeneous wave whose amplitude decays exponentially with distance away from the interface, is $\alpha_{inc} = 24.5$ deg. (respectively, $\alpha_{inc} = 57.4$ deg.). For solving the scattering problem, we chose the summation limits N_1 and N_2 over n such that $N_1 = -10$ and $N_2 = 10$ (all the propagating orders are taken into account in the calculations and the relations 15 are well satisfied). Figure 1 shows that the amount of S-diffracted energy, and also the amount of P-reflected energy, increase with increasing percentage of cracks at the interface. Consequently, neglecting the diffracted waves arising from the crack array at the interface leads to local underestimation of the actual amplitude of the reflected P-waves.

The most striking result is illustrated in Figure 2 which represents the computation of the first member of eq.19 as a function of the incidence angle α_{inc}. We can note that the energy conservation law is not satisfied for the different crack distributions (except for the case of 1% of cracks which is not shown here for the sake of brevity), contrary to the case of an interface between identical media [4]. One explanation to this striking result could be that the energy flux interferences cannot be neglected in such a configuration because, although the media in contact are elastic, the interface is laterally heterogeneous and not uniformly welded. Another explanation could be provided by analogy with studies in Non Destructive testing or in Acoustics. The spatial distribution of the welded contact areas and cracks at the interface can be viewed as a comb transducer which is commonly used for excitation and detection of surface waves. A part of the incident P-wave energy can be transferred to an interface wave (IW) which propagates along the interface between dissimilar elastic media. The IW can then be scattered by the crack distribution and recombined coherently into P-waves propagating in the bulk media. The part of the energy associated to the SW transferred to the bulk waves must then be taken into account in the energy conservation law 19, which will be done in future works.

§5. Conclusion

In the present paper, we have investigated the behaviour of time-harmonic elastic waves in the neighbourhood of a plane interface between dissimilar elastic media, where a periodic array of in-plane air-filled cracks exists. The aim of the work was to quantify the amount of incident wave energy distributed among the propagating scattered waves. The wave-scattering problem was formulated making use of the fundamentals of the theory recently developed by Danicki. Although in essence his method remains valid, the periodic array of cracks at the boundary between dissimilar media needed reformulation of the problem. The scattered fields were expressed in terms of Fourier series with coefficients depending on Legendre functions. Energies associated with the scattered waves of zeroth- and minus-first-diffraction order have been evaluated as a function of the incidence angle and for different crack distributions at the interface. We have shown that the amount of P-reflected energy (associated with zeroth-diffraction order) and the amount of S-reflected energy (associated with minus-first-diffraction order) increase with increasing percentage of cracks at the interface. As a result, neglecting the crack array leads to local underestimation of the actual amplitude of the reflected P-waves. We also suggest that, contrary to the case of an interface between identical media, the energy conservation law cannot be applied in this work since phenomena associated with interface wave scattering are not taken into account.

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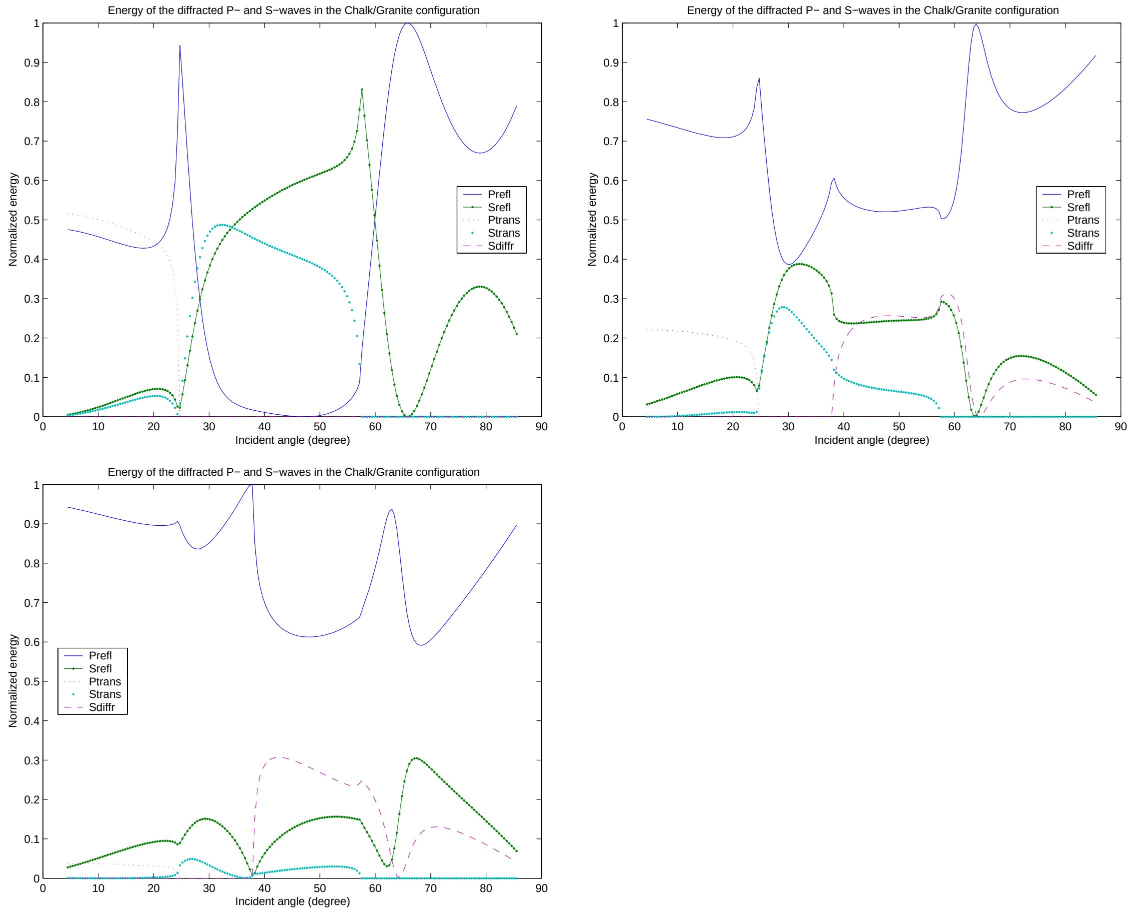

Figure 1: Energy associated with the different diffraction orders in the chalk and granite materials as a function of the incidence angle and for different crack distributions at the interface (1% of cracks (top left), 50% of cracks (top right), 75% of cracks (bottom left)).

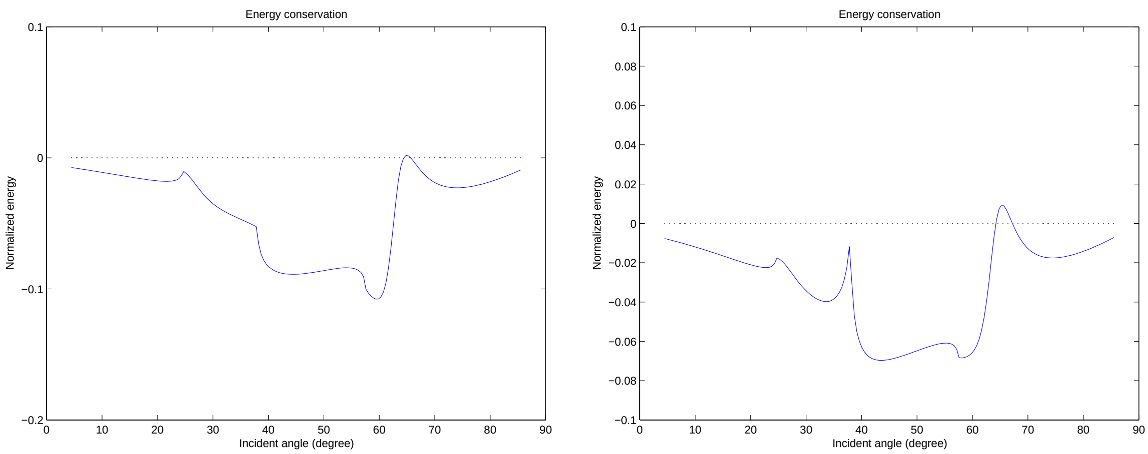

Figure 2: Computation of the first member of eq.19 as a function of the incidence angle and for two crack distributions at the interface chalk/granite (50% of cracks (left), 75% of cracks (right)).

Nathalie Favretto-Cristini
Centre National de la Recherche Scientifique
Laboratoire d'Imagerie Géophysique (FRE 2639)
Université de Pau et des Pays de l'Adour
BP 1155, 64013 Pau Cedex, France
nathalie.favretto@univ-pau.fr