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Multiphase flows in a sedimentary basin

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Abstract

In this work, the main phenomena of genesis and hydrocarbons migration in a sedimentary basin are presented. Then, a mathematical modelling of these characters is formulated. Pressure, hydrocarbons masses and water mass are the main unknowns which have held our attention for mathematical modelling. Phases miscibilities effects are also taken into account. Afterwards a numerical method to compute the mathematical model is described. The numerical method must be robust, stable and low cost in computing time. At last, numerical results are exposed in a black-oil flow case.

Keywords: sedimentary basin, mass formulation, hydrocarbons
AMS Classification: 65M,76T30

1 Introduction

Modelling of sedimentary basins allows to relate the history of the hydrocarbons from their genesis. It takes into account many physical and geological phenomena as erosion, compaction, rifts process, cracking chemical reactions, and so on ... This modelling is essential in petroleum engeneering as it allows in particular an effective selection of drilling sites. Other domains are also covered by this modelling as ground waters study and pollution risks.

In a first section, the main in a sedimentary basins contributing phenomena are described in a general way in order to suggest a mathematical modelling. This modelling is shown in the second section, where the various in the model chosen equations are setted, with the approach explanation. Once the model set, the time and space discretisation general principle is described along with details on the searched objective and the choosen approach. At last, a numerical essay is run to illustrate our simulator well doing in a black-oil flow.

2 Presentation of physical contributing phenomena in a sedimentary basin

A basin is a wide porous medium (a hundred kilometers in length and width, five kilometers in depht) in which hydrocarbons flow and store. The simulator objective is to relate the creation history -also called genesis-, the migration and the storage of these hydrocarbons along years which are counted in millions so as to locate better areas where hydrocarbons are concentrated.
To do that, the basin formation history relates the sedimentation process -rock and organics material deposits- which allows to create the geological layers. These layers hold organic materials which, under the heat and pressure action, are degraded into oil and gas. This chemical process is called cracking. More, these new made layers, are sinking into the earth along years due to the depositing sediment weight effect. The layers characteristics as porosity, absolute permeability are then evolving. These geological parameters allow to reproduce the compaction process in equations through their decreasing -compaction- or increasing -erosion- values. While this is doing, cracking appeared fluids are migrating up to the basin top.
The main process which allows these fluids migration to the top is the pressure difference which exists in the basin. The capillarity and gravity process take place also in this fluids migration, but are less significant. At least, during these migrations, interactions take place between fluids, and also components exchanges between phases -transfer of oil components into gas phase and gas components into oil phase- giving change into fluids properties.

3 Phenomena mathematical modelling

Two physical phenomenons are important when setting the sedimentary basin simulation, compaction and fluids flow. These two phenomenons setting equations are now described quickly. As compaction is a hard process to understand, a simple case is setted. A sedimention speed is supposed setted (which schemes the rock deposits), that allows to know the sediments weight that are added. This weight is called total burial and is recorded σ_T. These sediments are then packing down the beneath layers, that reduces the interstices and increases the pressure pore which is recorded P. Obviously, the erosion will produce an inverse effect and be modelled by a negative sedimentation speed. The effective constraint, recorded σ_e, simulates the intersices decreasing (or swelling). This three parameters relation is setted next:

$$\sigma_T = \sigma_e + P.$$

Then the porosity is involved next:

$$\Phi(\sigma_e) = \Phi_r + \Phi_a e^{-\frac{\sigma_e}{\sigma_a}} + \Phi_b e^{-\frac{\sigma_e}{\sigma_b}}$$

where $\Phi_r, \Phi_a, \Phi_b, \sigma_a, \sigma_b$ are constant given parameters for all geological layers.
The fluids migration modelling in a sedimentary basin is driven by three assumptions:

1. mass balance for every component
2. velocity behaviour law for every phase
3. thermodynamic equilibrium law

The mass balance law for every component j ($j = 1, ..., N$) is written next:

$$\frac{\partial m_j}{\partial t} + div\left(\sum_{\alpha=1}^{N_p} C_j^\alpha \rho_\alpha \vec{U}_\alpha\right) = q_j \; j = 1, ..., N$$

The filtration velocity $\vec{U}_\alpha$ appears in these equations. The law which models this velocity, must take into account porous medium, fluids interactions and gravity. The Darcy's law which results from experimental measures, has been kept. It formulates as following:

$$\vec{U}_\alpha = -\underline{\underline{K}} \frac{kr_\alpha}{\mu_\alpha} \left(\vec{\nabla} P_\alpha - \rho_\alpha \vec{g}\right).$$

At last, the thermodynamics equilibria laws allow components allocation in every phase. The sharing coefficient concept recorded β_j^α is introduced. This coefficient quantifies the component j in the phase α. These relations are written as following:

$$\Phi \rho_\alpha S_\alpha = \sum_{j=1}^{N} \beta_j^\alpha m_j \quad \alpha = 1, ..., N_p.$$

At least, the porous volume conservation law is introduced next:

$$\sum_{\alpha=1}^{N_p} S_\alpha = 1.$$

To study the mathematical model, simplifications and hypothesis are introduced. We suppose that water component is not present in any hydrocarbon phase and hydrocarbons components do not dissolve in water phase (so phase and component could be identificated). These hypotheses involve the next conditions:

$$\begin{aligned} C_j^w &= \beta_j^w = 0 \quad j = 1, ..., N-1 \\ C_w^w &= \beta_w^w = 1 \\ C_w^\alpha &= \beta_w \alpha = 0 \quad \alpha = 1, ..., N_p - 1 \end{aligned}$$

The capillary pressure is negligected too, so the pressure is the same for all phases. Lastly, three hydrocarbons components, two hydrocarbons phases and a water phase are simulated.
The study model is:

$$\begin{cases} \dfrac{\partial m_w}{\partial t} - div\left\{\underline{\underline{K}}\dfrac{kr_w}{\mu_w}\rho_w\Big(\vec{\nabla}P - \rho_w\vec{g}\Big)\right\} = 0, \\ \dfrac{\partial m_j}{\partial t} - div\left\{\underline{\underline{K}}\displaystyle\sum_{\alpha=1}^{N_p-1} C_j^\alpha \dfrac{kr_\alpha}{\mu_\alpha}\rho_\alpha\Big(\vec{\nabla}P - \rho_\alpha\vec{g}\Big)\right\} = q_j \quad j = 1,2,3, \\ \displaystyle\sum_{\alpha=o,g,w} S_\alpha = 1, \\ \Phi\rho_\alpha S_\alpha = \displaystyle\sum_{j=1}^{3}\beta_j^\alpha m_j \quad \alpha = o,g. \end{cases}$$

with the unknowns $P, m_1, m_2, m_3, S_o, S_g, S_w$.

Remark 1 *The water mass is eliminated from the equation by the relation* $m_w = \Phi\rho_w S_w$.

4 Time discretisation

The studied equations are stongly coupled and nonlinears. The current methods solve them with a Newton method considering the thermodynamic equilibrium in every time step. These modules which can calculate these thermodynamic equilibria are very sophisticated and costly in computing time.
The goal of our time discretisation is to split the thermodynamic modul from the transport equation. The hypothesis is to solve the transport equations in a default of thermodynamic equilibrium. First, the cracking (source term) is evaluated. Afterwards, thermodynamic parameters (viscosity, $\mu_\alpha^{n+\frac{1}{2}}$, density, $\rho_\alpha^{n+\frac{1}{2}}$ and so ...) are calculated. These equations which are strongly coupled through pressure, saturations and masses, could not be calculated easily. To uncoupled the equations, the IMPES scheme (IMPlicit Pressure Explicit Saturations) [1, 2] is used. The principle of this scheme is to consider all quantities depending on pressure as implicit terms [6] and the others quantities depending on masses and saturations as explicit terms. Masses and saturations are splitted from pressure. So,

the time discretisation of the system is:

$$\begin{cases} m_j^{n+1} = m_j^{n+\frac{1}{2}} + q_j^{n+\frac{1}{2}} \\ m_w^{n+1} = m_w^n + \delta t div\left(\underline{\underline{K^n}} \dfrac{kr_w^n}{\mu_w^n} \rho_w^n \left(\vec{\nabla} P^{n+1} - \rho_w^n \vec{g}\right)\right) = 0, \\ m_j^{n+1} = m_j^{n+\frac{1}{2}} + \delta t div\left(\underline{\underline{K^n}} \sum_{\alpha=1}^{N_p-1} (C_j^\alpha)^{n+\frac{1}{2}} \dfrac{kr_\alpha^n}{\mu_\alpha^{n+\frac{1}{2}}} \rho_\alpha^{n+\frac{1}{2}} \left(\vec{\nabla} P^{n+1} - \rho_\alpha^{n+\frac{1}{2}} \vec{g}\right)\right) \quad j = 1, 2, 3, \\ \sum_{\alpha=o,g,w} S_\alpha^{n+1} = 1, \\ \Phi^{n+1} \rho_\alpha^{n+\frac{1}{2}} S_\alpha^{n+1} = \sum_{j=1}^{3} (\beta_j^\alpha)^{n+\frac{1}{2}} m_j^{n+1} \quad \alpha = o, g. \end{cases}$$

The pressure equation is obtained through the masses and saturations elimination from these equations. This pressure is calculated from the pressure equation and then masses and saturations are deduced easily. A time step is shown in figure 1.

5 Numerical results

Numerical results of a sedimentary basin simulation are presented in figure 2. It is a two dimensions square basin $[0, 5000] \times [-5000, 0]$ with three geological layers. There is a sand layer (drain) dividing two clay layers. The hydrocarbon fluids are generated by cracking in the bottom of the basin in the zone $[0, 1000] \times [-5000, -4000]$. The time simulation is a hundred millions of years. A black-oil model (the light component is present in gas phase and oil phase, the heavy component stayes only in the oil phase) is simulated here. The hydrocarbon fluids which have been generated by cracking, flow vertically up to the top of the basin with few dispersion laterally. When they meet the drain, they speed up their migration and they are stopped by the impermeable clay layer. They fill up the drain and flow laterally and go to store in the side of the basin called reservoir.

6 Annexes

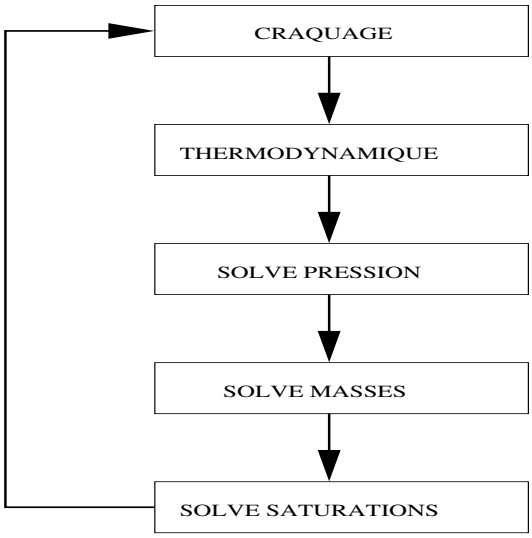

Figure 1: A time step algorithm resolution

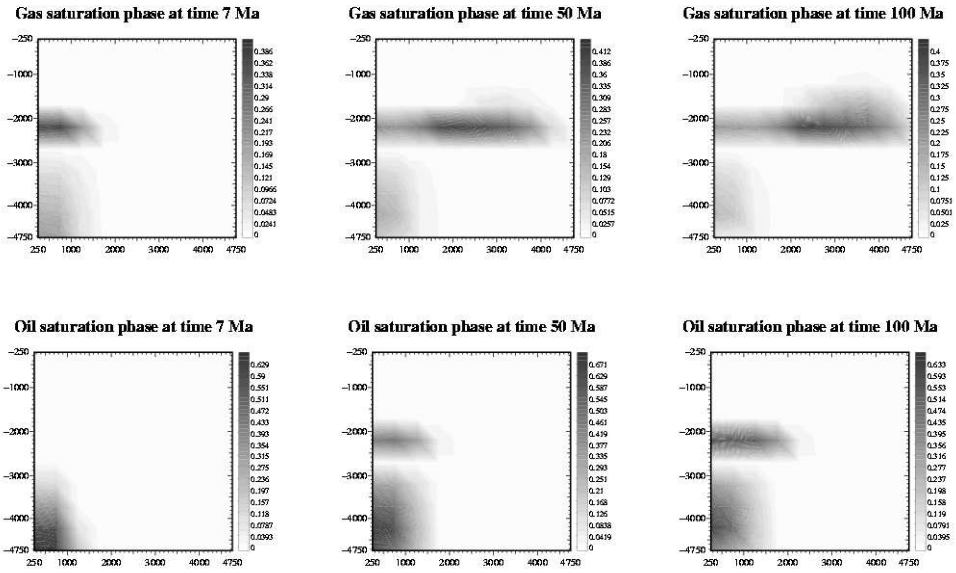

Figure 2: Oil phase saturations and gas phase saturations at $7Ma$, $50Ma$, $100Ma$

7 Nomenclature

- N_p number of hydrocarbon phases and water
- N number of hydrocarbon component and water component
- m_j component mass j
- C_j^α mass fraction of component j in the phase α
- β_j^α sharing coefficient of component j in the phase α
- $\vec{U}_\alpha$ phase filtration velocity α
- q_j source term modelling cracking
- Φ porosity
- $\underline{\underline{K}}$ absolute permeability
- kr_α relative permeability α
- μ_α phase viscosity α
- ρ_α phase density α
- P_α phase pressure α
- P global pressure or pore pressure
- Pc_α capillary pressure
- $\vec{g}$ gravity vector
- S_α phase saturation α
- w subscript representing phase or component water
- o subscript representing phase oil
- g subscript representing phase gas

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