Monografías del Seminario Matemático García de Galdeano **33**, 115–119 (2006)

GEOMETRIC MODELLING OF POLARIMETRIC TRANSFORMATIONS

C. Ferreira, I. San José, J. J. Gil and J. M. Correas

Abstract. A new method for analyzing the polarimetric properties of material samples, based on geometric representations of the transformations of the Poincaré sphere, is presented. Two transformed geometric objects are considered on the basis of the intensity (I) and the degree of polarization (P) of the transformed Stokes vectors. The "*I-Image*" of Poincaré sphere describes the diattenuation properties whereas the "*P-Image*" describes the depolarization properties. Birefringence properties are related to rotations arising in both objects. This model provides useful methods for analyzing experimental and industrial polarimetric measurements.

Keywords: Poincaré sphere, polarized light, Mueller matrix.

AMS classification: 00A79, 78A97.

§1. Introduction

Optical polarimetry is an important tool to extract information about several kind of material samples in scientific and industrial environments.

A complete information of the polarimetric properties is given by the 16 elements of the corresponding Mueller matrix.

In order to make easier the operation of the optical polarimeters and the sample analysis, we present a visual method to characterize and classify the samples under measurement. In other words, we look for a simple, efficient and easy-to-use tool, for people who work in polarimetry without any relation in maths.

From the concept of Poincaré sphere we define two new geometric objects which determine univocally the polarimetric properties of the sample and are useful as a method for easy operation of polarimeters.

Every state of polarization is represented in the Poincaré sphere taking the vectorial part defined by the three last components of the normalized Stokes vector [4]:

$$s = \begin{pmatrix} 1 \\ G\cos 2\varphi \cos 2\chi \\ G\cos 2\varphi \cos 2\chi \\ G\sin 2\varphi \end{pmatrix},$$

where G is the degree of polarization, φ the azimuth and χ the ellipticity.

Given a material sample characterized by its Mueller matrix M, the incident stokes vectors s are transformed to the emergent stokes vectors s' through the linear map $s' = Ms$.

§2. Image polarimetric objects

The *I-image* object corresponding to a Mueller matrix M is defined as the solid object given by the following points

$$\frac{s'_0}{\sqrt{{s'_1}^2+{s'_2}^2+{s'_3}^2}}\begin{pmatrix}s'_1\\ s'_2\\ s'_3\end{pmatrix},$$

whereas the *P-image* object is defined as the solid object given by the following points

$$\frac{1}{s'_0}\begin{pmatrix}s'_1\\ s'_2\\ s'_3\end{pmatrix}.$$

From the above definitions it is easy to see that the *I-image* object gives us direct information about diattenuation phenomena which could be appreciated in the asymmetric deformation of the surface object. As a degenerated case includes isotropic attenuators that present size but not shape changes.

On the other hand, the *P-image* object give us direct information about depolarization (surface deformation) i.e. its surface shape is preserved by all pure systems.

§3. Physical polarimetric behaviours

All the possible physical polarimetric behaviour are covered by the following cases:

1. Pure diattenuators

   In this case we observe deformation of the *I-image* object, whereas the surface shape of the *P-image* object is preserved. Notice that the Mueller matrix is symmetric.

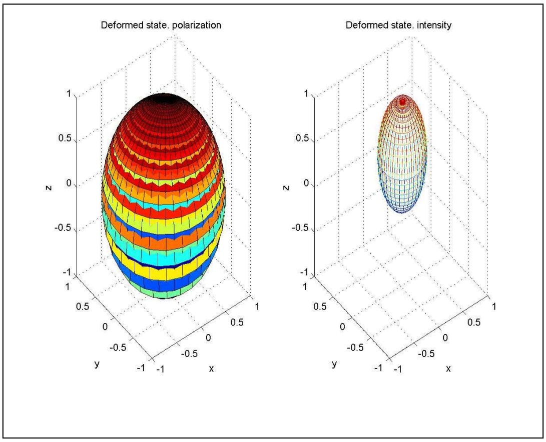

Figure 1: Transformed states for a pure diattenuator sample

2. Pure retarders

   None of the objects are deformed. We have a rotation of both of them. Mueller matrix is a rotation matrix.

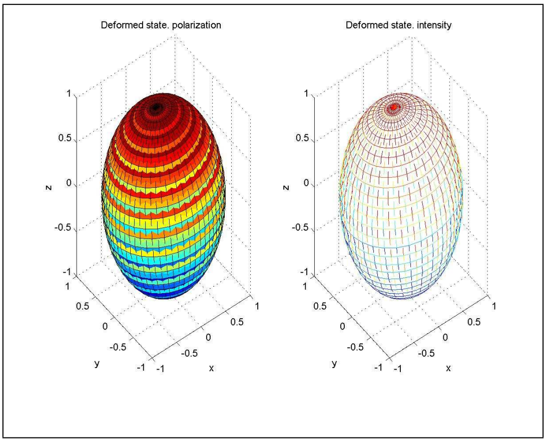

Figure 2: Transformed states for a pure retarder

3. Parallel combination of pure elements (depolarizers)

   The *P-image* object is deformed. Depending on the statistical mixture of the sample we have

   (a) Statistical mixture of diattenuators

       The *I-image* object is also deformed. The Mueller matrix is symmetric.

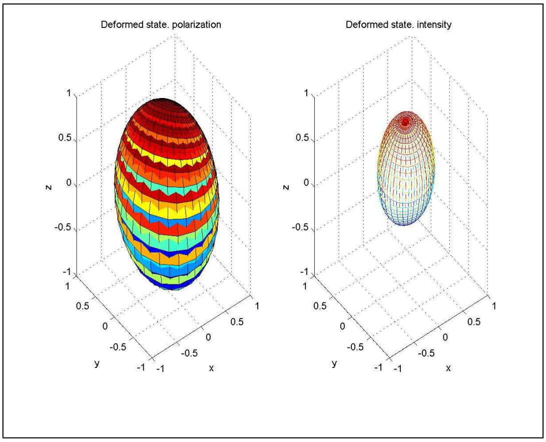

Figure 3: Transformed states for a statistical mixture of diattenuators

(b) Statistical mixture of retarders
The *I-image* object is not deformed.

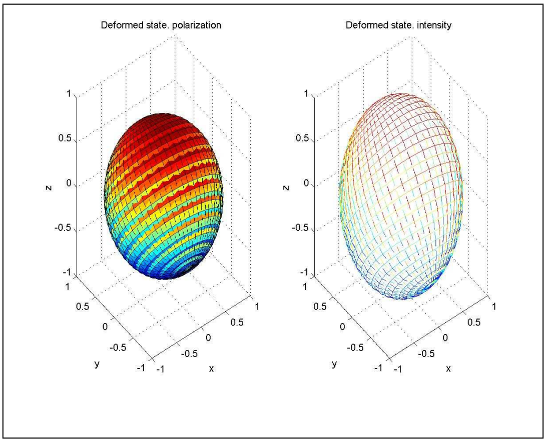

Figure 4: Transformed states for a statistical mixture of retarders

(c) Statistical mixture of diattenuators and retardes.
Both objects are deformed. The Mueller matrix is not symmetric.

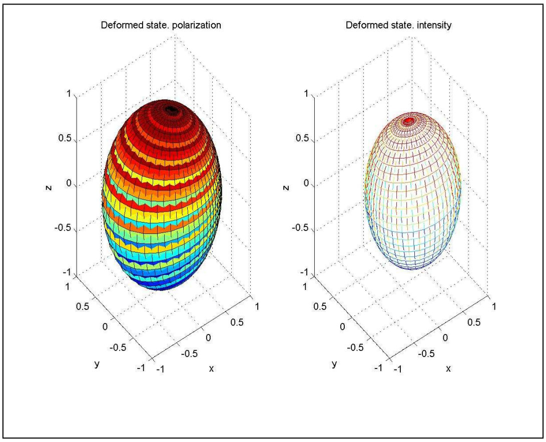

Figure 5: Transformed states for a statistical mixture of diattenuators and retardes

Acknowledgements

This paper has been carried out in the context of the research line supported by grant DGA Po64/2000 of Diputación General de Aragón. The authors are members of the "Grupo de óptica matemática" (GOM), Universidad de Zaragoza.

References

[1] B.DEBOO, J.SASIAN AND R.CHIPMAN Degree of polarization surfaces and maps for analysis of depolarization. *Optics Express 4941*, 12, (2004), 49414958.

[2] GIL, J.J. Characteristic properties of Mueller matrices. *Journal Optical Society of America A 17* (2000), 328–334.

[3] GIL, J. J., AND BERNABEU, E. Obtainment of the polarizing and retardation parameters of a non-depolarizing optical system from the polar decomposition of its Mueller matrix. *Optiks 76* (1987), 67–71.

[4] GIL, J. J., AND ARNAL, P. M. El modelo de la esfera de Poincaré: representación de medios materiales. *Zubía. I.E.R. 9* (1991), 9–17.

[5] MELERO, P. A. *Modelado matemático de las propiedades polarimétricas de la luz y de los medios materiales*. Ph. D. Thesis. Facultad de Ciencias. Universidad de Zaragoza, 2003.

[6] GIL, J. J., CORREAS, J. M., MELERO, P., AND FERREIRA, C. Generalized polarization algebra. *Monogr. Semin. Mat. García Galdeano 31* (2004), 161–167.

C. Ferreira, J. M Correas
Departamento de Matemática Aplicada
Universidad de Zaragoza
jcorreas@unizar.es, and cferrei@unizar.es

I. San José
Instituto Aragonés de Estadística
isanjose@aragon.es

J. J. Gil
ICE
Universidad de Zaragoza
ppgil@unizar.es