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SYSTEMS OF SCHRÖDINGER EQUATIONS: POSITIVITY AND NEGATIVITY

Bénédicte Alziary, Jacqueline Fleckinger
and Marie-Hélène Lécureux

Abstract. We consider here Schrödinger operator $-\Delta + q(x)\bullet$ defined in the entire space $\mathbb{R}^N$, with a potential q tending to $+\infty$ at infinity with a sufficiently fast growth. The ground state positivity and negativity for a Schrödinger equation with spectral parameter says that, if the spectral parameter is lower than the principal eigenvalue, the solutions satisfy ground state positivity (greater than a positive constant times the ground state) and if the spectral parameter is slightly greater than the principal eigenvalue, then the solutions satisfy ground state negativity (lower than minus a positive constant times the ground state). We extend this ground state positivity and negativity to cooperative and noncooperative systems of two Schrödinger equations.

Keywords: Positive or negative solutions, pointwise bounds, ground state, principal eigenvalue, cooperative and noncooperative systems.

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§1. Introduction

Positivity or negativity of weak L^2-solutions of a linear partial differential equation with the Schrödinger operator,

$$-\Delta u + q(x)u - \lambda u = f(x) \quad \text{in } \mathbb{R}^N, \tag{1}$$

has been a subject of a number of research articles and monographs, see e.g. Alziary, Fleckinger and Takáč [3, 5], Alziary and Takáč [2], and many others. Here, f is a given function satisfying $0 \leq f \not\equiv 0$ in $\mathbb{R}^N$ ($N \geq 1$), and λ stands for the spectral parameter. Let φ_1 denote the positive eigenfunction associated with the principal eigenvalue λ_1 of the Schrödinger operator $\mathscr{A} = -\Delta + q(x)\bullet$ in $L^2(\mathbb{R}^N)$. Assume that the potential $q(x)$ is radially symmetric and grows fast enough near infinity, and f is a “sufficiently smooth” perturbation of a radially symmetric function, $f \not\equiv 0$ and $0 \leq f/\varphi \leq C \equiv$ const a.e. in $\mathbb{R}^N$. For such equation (1), it is possible to show that u satisfies the ground state positivity for $-\infty < \lambda < \lambda_1$ (i.e., $u \geq c\varphi_1$ with $c \equiv$ const > 0) and satisfies the ground state negative for $\lambda_1 < \lambda < \lambda_1 + \delta$ (i.e., $u \leq -c\varphi_1$ with $c \equiv$ const > 0), where $\delta > 0$ is a number depending on f. The constant $c > 0$ depends on both λ and f.

In their book, Protter and Weinberger [12] give a maximum principle for weakly coupled systems of essentially positive elliptic equations. Then several authors revisited the problem in the case of a bounded domain, De Figueiredo and Mitidieri [10], Mitidieri and Sweers [11] and Cosner and Schaefer [9] for the maximum principle. The anti-maximum, always for bounded domain, was studied in particular by Sweers [13] and Takáč [14]. For a system of Schrödinger equations on the whole space, Abakhti-Mchachti and Fleckinger [6] or Alziary, Cardoulis and Fleckinger [1] obtained the maximum principle for a cooperative system but not the ground state positivity. Alziary, Fleckinger and Takáč [4] proved the ground state positivity for a cooperative system and for a (2×2) noncooperative system by inserting the (2×2) noncooperative system into a (3×3) cooperative one. Recently Besbas [7] gave a result concerning ground state negativity for particular cooperative system. Note that all those results are obtained for radially symmetric potential.

Here our purpose is to show, on a (2×2) systems of Schrödinger equations in the whole space $\mathbb{R}^N$, how to obtain ground state positivity and negativity for cooperative and noncooperative system. We consider the following system :

$$\mathscr{L}\begin{pmatrix}u\\ v\end{pmatrix}=\begin{pmatrix}-\Delta+q(x)\bullet & 0\\ 0 & -\Delta+q(x)\bullet\end{pmatrix}\begin{pmatrix}u\\ v\end{pmatrix}=\begin{pmatrix}\lambda u+au+bv+f\\ \lambda v+cu+dv+g\end{pmatrix}. \tag{2}$$

The functions f and g are in $L^2(\mathbb{R}^n)$ and λ is a spectral parameter. The coefficients a,b,c,d are constant and we denote by $M=\left(\begin{smallmatrix}a & b\\ c & d\end{smallmatrix}\right)$. If $b\geq 0$ and $c\geq 0$, the system is called cooperative. Instead of inserting the (2×2) noncooperative system into a (3×3) cooperative one,the idea is to use for both cooperative and noncooperative systems the decomposition of the resolvant $(\lambda I-\mathscr{L})^{-1}$ for λ near λ_1.

This article is organized as follows. In Section 2 we give some notations and definitions and we state our main result, Theorem 1. In Section 3 we first recall the result for the single equation. Indeed the proof of the theorem 1 will use the ground state positivity and negativity for one equation. Finally in Section 4, we give the proof of our main result.

§2. Main Result

The Schrödinger operator $\mathscr{A}$ denotes the selfadjoint extension of the symmetric operator in $L^2(\mathbb{R}^n)$ defined by

$$\mathscr{A}u=-\Delta u+q(x)u \quad \text{for } x\in\mathbb{R}^n \quad \text{and } u\in C_0^2(\mathbb{R}^n).$$

The potential $q\in L^\infty_{\mathrm{loc}}(\mathbb{R}^n)$, tending to infinity when $|x|$ goes to infinity, is supposed to be greater than some positive constant, $0<\mathrm{Cst}\leq q(x)$. With such hypotheses on the potential, the spectrum of $\mathscr{A}$ consists on a sequence of positive eigenvalues tending to infinity. The smallest one, λ_1 is given by the Rayley quotient

$$\lambda_1=\inf_{u\in V_q(\mathbb{R}^n)}\left\{\int_{\mathbb{R}^n}|\nabla u|^2\,dx+\int_{\mathbb{R}^n}q(x)|u|^2\,dx,\ \text{ with } \ \|u\|_{L^2(\mathbb{R}^n)}=1\right\},$$

where the weighted space V_q is defined as follows:

$$V_q(\mathbb{R}^n)=\left\{u\in L^2(\mathbb{R}^n):\int_{\mathbb{R}^n}|\nabla u|^2\,dx+\int_{\mathbb{R}^n}q(x)|u|^2\,dx<\infty\right\}.$$

This principal eigenvalue λ_1 is associated with a positive eigenfunction $\varphi_1>0$ normalized by $\|\varphi_1\|^2_{L^2(\mathbb{R}^n)}=1$. This positive eigenfunction $\varphi_1>0$ is called the ground state. The domain of the operator $\mathscr{A}$ is denoted by

$$\mathscr{D}(\mathscr{A}) = \{u \in V_q(\mathbb{R}^n) : (-\Delta + q)u \in L^2(\mathbb{R}^n)\}.$$

Finally, let us recall the definition of the ground state positivity and negativity, introduced by Alziary, Fleckinger and Takáč [2, 3]

Definition 1. A function $u \in L^2(\mathbb{R}^N)$ satisfies the *ground state positivity* if there exists a constant $c > 0$ such that

$$u \geq c\,\varphi_1 \quad \text{almost everywhere in } \mathbb{R}^N.$$

Analogously, $u \in L^2(\mathbb{R}^N)$ satisfies the *ground state negativity* if there exists a constant $c > 0$ such that

$$u \leq -c\,\varphi_1 \quad \text{almost everywhere in } \mathbb{R}^N.$$

The ground state positivity (or ground state negativity) of a sufficiently smooth solution u to the equation (1), for $\lambda < \lambda_1$ (or $\lambda_1 < \lambda < \lambda_1 + \delta$, respectively), is an important result with numerous applications to both linear and nonlinear elliptic problems in $\mathbb{R}^N$, see Alziary and Takáč [2]. Here, δ is a positive number depending upon f.

Those results are similar to the maximum or anti-maximum principle in a bounded domain $\Omega \subset \mathbb{R}^N$, $N \geq 1$, which have been established in the work of Clément and Peletier [8], Sweers [13] and Takáč [14]. But the case of the Schrödinger operator on $\Omega = \mathbb{R}^N$ is more difficult; the hypothesis $f \in L^p(\Omega)$ $(p > N)$ is no longer sufficient. We need to take a smaller space for f, namely, a strongly ordered Banach space X introduced in Alziary and Takáč [2]:

$$X = \{u \in L^2(\mathbb{R}^N) : \ u/\varphi_1 \in L^\infty(\mathbb{R}^N)\}, \tag{3}$$

endowed with the ordered norm

$$\|u\|_X = \inf\{C \in \mathbb{R} : \ |u| \leq C\varphi_1 \text{ almost everywhere in } \mathbb{R}^N\}. \tag{4}$$

The ordering "$\leq$" on X is the natural pointwise ordering of functions. This means that X is an ordered Banach space whose positive cone X_+ has nonempty interior $\mathring{X}_+$.

We denote by (r,x') the spherical coordinates in $\mathbb{R}^N$, that is, $x = rx' \in \mathbb{R}^N$, where $r = |x|$ and $x' = r^{-1}x \in \mathbf{S}^{N-1}$ if $x \neq 0$; we set $r = 0$ and leave $x' \in \mathbf{S}^{N-1}$ arbitrary if $x = 0$. As usual, $\mathbf{S}^{N-1}$ denotes the unit sphere in $\mathbb{R}^N$ centered at the origin. We refer to r and x' as the radial and azimuthal variables, respectively. The surface measure on $\mathbf{S}^{N-1}$ is denoted by σ; we let $\sigma_{N-1} = \sigma(\mathbf{S}^{N-1})$ be the surface area of $\mathbf{S}^{N-1}$.

For any $\alpha > 0$, we introduce the Banach space $X^{\alpha,2}$ of all functions $f \in L^2_{\text{loc}}(\mathbb{R}^N)$ having the following properties:

$$\big[(-\Delta_S)^{\alpha/2} f\big](r,\bullet) \in L^2(\mathbf{S}^{N-1}) \quad \text{for all } \ r > 0,$$

where Δ_S denotes the Laplace-Beltrami operator on the sphere $\mathbf{S}^{N-1}$, and there is a constant $C \geq 0$ such that, for almost every $r > 0$,

$$\frac{1}{\sigma_{N-1}}\int_{\mathbf{S}^{N-1}} |f(r,x')|^2\, d\sigma(x') + \frac{1}{\sigma_{N-1}}\int_{\mathbf{S}^{N-1}} \left|[(-\Delta_S)^{\alpha/2} f](r,x')\right|^2 d\sigma(x') \leq [C\varphi_1(r)]^2.$$

The smallest such constant C defines the norm $\|f\|_{X^{\alpha,2}}$ in $X^{\alpha,2}$. Notice that, for $f(x) \equiv f(|x|)$, we have $f \in X^{\alpha,2} \iff f \in X$ together with the norms $\|f\|_{X^{\alpha,2}} = \|f\|_X$. Furthermore, if $\alpha > \frac{N-1}{2}$ then $X^{\alpha,2}$ is continuously imbedded into X, by the Sobolev imbedding theorem for $W^{\alpha,2}(\mathbf{S}^{N-1}) \hookrightarrow C(\mathbf{S}^{N-1})$. Of course, the Hilbert space $W^{\alpha,2}(\mathbf{S}^{N-1})$ is defined to be the domain of $(-\Delta_S)^{\alpha/2}$ in $L^2(\mathbf{S}^{N-1})$ endowed with the graph norm.

Taking $N \geq 2$, we establish the ground state positivity and negativity for f and g from the Banach space $X^{\alpha,2}$. The necessity of such a restriction for the Schrödinger operator in $L^2(\mathbb{R}^N)$ has been discussed and partly justified in [3, Remark 2.1 and Lemma 2.2] and in [4, Example 4.1].

In order to formulate our hypothesis on the potential $q(x)$, $x \in \mathbb{R}^N$, we first introduce the following class of auxiliary functions $Q(r)$ of $r \equiv |x|$, $R_0 \leq r < \infty$, for some $R_0 > 0$:

$$\begin{cases} Q(r) > 0, \ Q \text{ is locally absolutely continuous,} \\ Q'(r) \geq 0, \text{ and } \int_{R_0}^{\infty} Q(r)^{-1/2}\, dr < \infty. \end{cases} \tag{5}$$

We assume that the potential q is radially symmetric, $q(x) \equiv q(|x|)$, $x \in \mathbb{R}^N$, where $q(r)$ is a Lebesgue measurable function satisfying the following hypothesis, with some auxiliary function $Q(r)$ which obeys (5):

$$\begin{cases} \text{The potential } q : \mathbb{R}_+ \to \mathbb{R} \text{ is locally essentially bounded, } q(r) \geq \\ \text{const} > 0 \text{ for } r \geq 0, \text{ and there exists a constant } c_1 > 0 \text{ such that} \\ c_1 Q(r) \leq q(r) \text{ for } R_0 \leq r < \infty. \end{cases} \tag{H}$$

We always suppose that M satisfies

$$\begin{cases} a > 0, \ d > 0, \ c \neq 0, \text{ and } a \geq d, \\ D = (a-d)^2 + 4bc > 0. \end{cases} \tag{$\mathrm{H_M}$}$$

Hypotheses $a > 0$, $d > 0$ and $a \geq d$ can always be satisfied by adding a constant times u in both sides of the first equation, a constant times v in both sides of the second equation and eventually switching the two equations to get $a \geq d$. But the matrix M must not have complex eigenvalues. So M has the two following eigenvalues:

$$\mu^+ = \frac{a+d+\sqrt{D}}{2} \quad \text{and} \quad \mu^- = \frac{a+d-\sqrt{D}}{2},$$

Let us, now, formulate our main result.

Theorem 1. *Let the hypotheses* (H) *and* ($\mathrm{H_M}$) *be satisfied. Assume that u and v are in $\mathscr{D}(\mathscr{A})$ and satisfy the system (2) with f et g in $X^{\alpha,2}$, for some $\alpha > \frac{N-1}{2}$, $f + \frac{2b}{(a-d)+\sqrt{D}} g \geq 0$ a.e. in $\mathbb{R}^N$ and $f + \frac{2b}{(a-d)+\sqrt{D}} g > 0$ in some set of positive Lebesgue measure.*

- *Before $\lambda_1 - \mu^+$:*
  there exists a positive number δ (depending upon f, g and M) such that, for every $\lambda \in (\lambda_1 - \mu^+ - \delta, \lambda_1 - \mu^+)$, inequalities

$$u \geq c_u \varphi_1 \ \text{ and } \ v \geq c_v \varphi_1 \quad \text{in } \mathbb{R}^N \ \text{ in the case } c > 0, \tag{6}$$

$$u \geq c_u \varphi_1 \ \text{ and } \ v \leq -c_v \varphi_1 \ \text{ in } \mathbb{R}^N \ \text{ in the case } c < 0, \tag{7}$$

  are valid with two constants $c_u > 0$ and $c_v > 0$ (depending upon f, g, M and λ).

- *After $\lambda_1 - \mu^+$:*
  there exists a positive number δ (depending upon f, g and M) such that, for every $\lambda \in (\lambda_1 - \mu^+, \lambda_1 - \mu^+ + \delta)$, inequalities

$$u \le -c_u\,\varphi_1 \;\; \text{and} \;\; v \le -c_v\,\varphi_1 \;\; \text{in} \;\; \mathbb{R}^N \;\; \text{in the case } c > 0, \tag{8}$$

$$u \le -c_u\,\varphi_1 \;\; \text{and} \;\; v \ge c_v\varphi_1 \quad \text{in} \;\; \mathbb{R}^N \;\; \text{in the case } c < 0, \tag{9}$$

are valid with two constants $c_u > 0$ and $c_v > 0$ (depending upon f, g, M and λ).

§3. Some known results for a single equation on the whole space

The following theorem was established by Alziary, Fleckinger and Takáč, first for $\mathbb{R}^2$ in [3] and then using Fourier series with spherical harmonics for $\mathbb{R}^N$ in [5].

Theorem 2. *Let the hypothesis* (H) *be satisfied. Assume that $u \in \mathscr{D}(\mathscr{A})$, $\mathscr{A}u - \lambda u = f \in L^2(\mathbb{R}^N)$, $\lambda \in \mathbb{R}$, and $f \ge 0$ a.e. in $\mathbb{R}^N$ with $f > 0$ in some set of positive Lebesgue measure. Then, for every $\lambda \in (-\infty, \lambda_1)$, there exists a constant $c > 0$ (depending upon f and λ) such that*

$$u \ge c\,\varphi_1 \quad \text{in} \;\; \mathbb{R}^N. \tag{10}$$

Moreover, if also $f \in X^{\alpha,2}$ for some $\alpha > \frac{N-1}{2}$, then there exists a positive number δ (depending upon f) such that, for every $\lambda \in (\lambda_1, \lambda_1 + \delta)$, the inequality

$$u \le -c\,\varphi_1 \quad \text{in} \;\; \mathbb{R}^N \tag{11}$$

is valid with a constant $c > 0$ (depending upon f and λ).

In fact, the proof of this result gives more precisions about the behaviour of the constant c when λ goes to λ_1. The next remark details how the constant depends upon f an λ.

Remark 1. For $\lambda < \lambda_1$, λ near λ_1, we have $u \ge C(f,\lambda)\varphi_1$, with $C(f,\lambda) = \frac{\int_{\mathbb{R}^n} f\varphi_1}{\lambda_1 - \lambda} - \Gamma(\lambda, f)$ and $\lim_{\lambda\to\lambda_1} \Gamma(\lambda, f) = \Gamma < \infty$. So when λ goes to λ_1, u becomes very large. By the strong maximum principle, we have also $|u| \le \frac{\|f\|_X}{(\lambda_1 - \lambda)}\varphi_1$.

For $\lambda > \lambda_1$, λ near λ_1, we get $u \le -C(f,\lambda)\varphi_1$, with $C(f,\lambda) = \frac{\int_{\mathbb{R}^n} f\varphi_1}{\lambda - \lambda_1} - \Gamma(\lambda, f)$ and $\lim_{\lambda\to\lambda_1} \Gamma(\lambda, f) = \Gamma < \infty$. So when λ goes to λ_1, $-u$ becomes very large. The proof of this remark is given in [7].

§4. Proof of the Theorem

Proof. The two eigenvectors v^+ and v^- associated respectively with the eigenvalues μ^+ and μ^- are

$$v^+ = \begin{pmatrix} \frac{a-d+\sqrt{D}}{2} \\ c \end{pmatrix} \quad \text{and} \quad v^- = \begin{pmatrix} -b \\ \frac{a-d+\sqrt{D}}{2} \end{pmatrix}.$$

So the system can be rewritten

$$\begin{cases} -\Delta\tilde{u} + q\tilde{u} = (\lambda + \mu^+)\tilde{u} + \tilde{f} & \text{in } \mathbb{R}^n \\ -\Delta\tilde{v} + q\tilde{v} = (\lambda + \mu^-)\tilde{v} + \tilde{g} & \text{in } \mathbb{R}^n \end{cases}$$

with

$$\begin{pmatrix}\tilde{u}\\ \tilde{v}\end{pmatrix} = P\begin{pmatrix}u\\ v\end{pmatrix}, \quad \begin{pmatrix}\tilde{f}\\ \tilde{g}\end{pmatrix} = P\begin{pmatrix}f\\ g\end{pmatrix} \quad \text{and} \quad P = \frac{1}{\sqrt{D}}\begin{pmatrix}1 & \frac{2b}{(a-d)+\sqrt{D}}\\ \frac{-2c}{(a-d)+\sqrt{D}} & 1\end{pmatrix}.$$

The two functions $\tilde{u}$ and $\tilde{v}$ are solutions of the two independent following equations:

$$-\Delta\tilde{u} + q\tilde{u} = (\lambda+\mu^+)\tilde{u} + \frac{1}{\sqrt{D}}\left(f + \frac{2b}{(a-d)+\sqrt{D}}\,g\right), \tag{12}$$

$$-\Delta\tilde{v} + q\tilde{v} = (\lambda+\mu^-)\tilde{v} + \frac{1}{\sqrt{D}}\left(\frac{-2c}{(a-d)+\sqrt{D}}\,f + g\right). \tag{13}$$

After solving those two equations, the initial functions u and v could be calculated by

$$u = \frac{(a-d)+\sqrt{D}}{2}\,\tilde{u} - b\tilde{v}, \tag{14}$$

$$v = c\tilde{u} + \frac{(a-d)+\sqrt{D}}{2}\,\tilde{v}. \tag{15}$$

We suppose $\lambda < \lambda_1 - \mu^-$, and so the equation (13) satisfies the maximum principle. The function f and g are in X and so for some constant $C_{\tilde{g}}$, we have

$$|\tilde{v}| \le (\lambda_1 - \lambda - \mu^-)^{-1} C_{\tilde{g}}\varphi_1. \tag{16}$$

- For $\lambda < \lambda_1 - \mu^+ < \lambda_1 - \mu^-$, the equation (12) satisfies the fundamental positivity, so we have

  $$|\tilde{v}| \le \frac{C_{\tilde{g}}}{\lambda_1 - \lambda - \mu^-}\varphi_1 \le \frac{C_{\tilde{g}}}{\mu^+ - \mu^-}\varphi_1 \quad \text{and} \quad \tilde{u} \ge C(\lambda,\tilde{f})\varphi_1,$$

  with $C(\lambda,\tilde{f})$ which goes to $+\infty$ when λ tends to λ_1.

  Consequently, $\tilde{v}$ stays bounded and $\tilde{u}$ becomes very large positive when λ goes to λ_1. So there exists a positive number δ (depending upon f, g and M) such that, for every $\lambda \in (\lambda_1 - \mu^+ - \delta, \lambda_1 - \mu^+)$, by (14) and (15), we get for u and v, in the case $c > 0$,

  $$u \ge c_u\varphi_1 \quad \text{and} \quad v \ge c_v\varphi_1, \quad c_u \text{ and } c_v \text{ are positive constants.}$$

  In that case, it is possible to show, using the Neumann series for the resolvent $(\lambda I - \mathscr{L})^{-1}$, that the ground state positivity is true for all $\lambda < \lambda_1 - \mu^+$.

  Of course, for $c < 0$, we have,

  $$u \ge c_u\varphi_1 \quad \text{and} \quad v \le -c_v\varphi_1, \quad c_u \text{ and } c_v \text{ are positive constants.}$$

- For $\lambda_1 - \mu^+ < \lambda < \lambda_1 - \mu^-$, the upper bound (16) stays valid and (12) satisfies the ground state negativity, so there exists $\delta_{\tilde{u}} \le \mu^+ - \mu^-$ such that for every $\lambda \in (\lambda_1 - \mu^+, \lambda_1 - \mu^+ + \delta_{\tilde{u}})$, we have

  $$|\tilde{v}| \le \frac{C_{\tilde{g}}}{\lambda_1 - \lambda - \mu^-}\varphi_1 \le \frac{C_{\tilde{g}}}{\mu^+ - \mu^- - \delta_{\tilde{u}}}\varphi_1 \quad \text{and} \quad \tilde{u} \le -C(\lambda,\tilde{f})\varphi_1,$$

with $C(\lambda, \tilde{f})$ which goes to $+\infty$ when λ tends to λ_1.

Consequently, $\tilde{v}$ stays bounded and $\tilde{u}$ becomes very large negative when λ goes to λ_1. So there exists a positive number δ (depending upon f, g and M) such that, for every $\lambda \in (\lambda_1 - \mu^+, \lambda_1 - \mu^+ + \delta)$, by (14) and (15) we get for u and v, in the case $c > 0$,

$$u \leq -c_u \varphi_1 \quad \text{and} \quad v \leq -c_v \varphi_1, \quad c_u \text{ and } c_v \text{ are positive constants.}$$

Of course, for $c < 0$, we have

$$u \leq -c_u \varphi_1 \quad \text{and} \quad v \geq c_v \varphi_1, \quad c_u \text{ and } c_v \text{ are positive constants.} \quad \square$$

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Bénédicte Alziary, Jacqueline Fleckinger and Marie-Hélène Lécureux
CEREMATH MIP UMR 5640
Université Toulouse 1
31042 TOULOUSE Cedex
`alziary@univ-tlse1.fr` and `jfleck@univ-tlse1.fr`