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Some approximation problems by generalized Wendland compactly supported radial basis functions

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SUMMARY

In a wide range of applications, positive kernels have proven to be very useful tools due to desirable properties such as naturally arise, reproducing kernel Hilbert space of Hilbert spaces of continuous functions, frequently appearing as radial basis functions

$$K(x, y) = \phi(\langle x - y \rangle_d), \quad \forall\, x, y \in \mathbb{R}^d,$$

where $\langle \cdot \rangle_d$ represents the Euclidean norm in $\mathbb{R}^d$, being $\phi : [0, +\infty) \to \mathbb{R}$ a smooth univariant function.

Usually, these kernels are not compactly supported or they are not differentiable. The Wendland functions have no such drawbacks[2]. They are polynomials in $[0, 1]$ of minimal degree $\lfloor d/2 \rfloor + 3k + 1$ and yield positive definite C^{2k} radial basis functions on $\mathbb{R}^d$. Moreover, they are reproducing kernels of Hilbert spaces isomorphic to the Sobolev space $H^{d/2+k+1/2}(\mathbb{R}^d)$. But this means that integer-order Sobolev spaces in even dimensions are not covered. Thus, it is necessary to extend the classical Wendland functions to the generalized Wendland functions[1] that allow to reproduct kernels of Hilbert spaces isomorphic to integer-order Sobolev spaces in even dimensions while holding the compact support property.

In this work we deal with interpolation and smoothing problems in a finite-dimensional generalized Wendland functions space; we prove the corresponding convergence results showing some graphical and numerical examples in $\mathbb{R}^2$.

Finally, we present a numerical method for solving 2D Fredholm integral equations of second kind by generalized Wendland radial basis functions.

Keywords: Generalized Wendland functions, interpolation, smoothing, 2D Fredholm integral equations

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