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Finite element analysis for a problem with the Ventcel boundary condition

Fabien Caubet[1], **Joyce Ghantous**[1], **Charles Pierre**[1]

SUMMARY

Let Ω be an open-bounded and connected domain of $\mathbb{R}^n$ (n=2,3) with $\Gamma = \partial\Omega$ as its compact smooth boundary. We define a finite element method for numerically approximating the solution of the following system:

$$\left\{ \begin{array}{rcll} -\Delta u + \kappa u & = & f & \text{in } \Omega, \\ -\beta \Delta_\Gamma u + \partial_{\mathrm{n}} u + \alpha u & = & g & \text{on } \Gamma, \end{array} \right.$$

where n is the outer unit normal vector on Γ, $f \in \mathrm{L}^2(\Omega)$ and $g \in \mathrm{L}^2(\Gamma)$, $\kappa \geq 0$, $\alpha > 0$ and $\beta > 0$ are constants. We discretize the domain Ω and we wish to compare the error between the solution of the exact problem $u \in \mathcal{H} = \mathrm{H}^1(\Omega) \cap \mathrm{H}^1(\Gamma)$ which we equipped with the norm $\|v\|_{\mathcal{H}} = \sqrt{\|v\|^2_{\mathrm{H}^1(\Omega)} + \|v\|^2_{\mathrm{H}^1(\Gamma)}}$ and the solution of the discrete formulation u_h defined on the approximated domain Ω_h. However each function is defined on a different domain, to overcome this problem we will estimate the error between the exact solution and the solution of the lifted problem using the transformation defined in [4], [3] and [2]. Denote u_h^ℓ the lift of u_h on Ω, then our main result is the following error estimate where we use a P^k finite element space ($k \geq 1$):

$$\|u - u_h^\ell\|_{\mathcal{H}} = O(h^k + h^{r+1}),$$

where r is the geometrical degree of approximation of Ω and h is the biggest diameter of a cell of the mesh. Finally we perform numerical simulations which validate this result.

Keywords: Laplace-Beltrami operator, Finite element method, lifted functions, error analysis, geometric error, eigenvalue and eigenvectors approximation.

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[1]Université de Pau et des Pays de l'Adour, E2S UPPA, CNRS, LMAP, UMR 5142, Pau, France