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A bifurcation result for a class of superlinear elliptic problems

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SUMMARY

We will discuss the existence of positive solutions of a class of superlinear elliptic equations in a bounded smooth domain $\Omega \subset \mathbb{R}^N$

$$-\Delta u = \lambda u + a(x) f(u), \quad \text{in } \Omega, \qquad u = 0, \quad \text{on } \partial\Omega, \tag{1}$$

where $\lambda \in \mathbb{R}$ is a real parameter, $a \in C^1(\bar{\Omega})$ **changes sign** in Ω,

$$f(s) := g(s) + h(s), \text{ with } h(s) := \frac{|s|^{2^*-2}s}{[\ln(e+|s|)]^{\alpha}}, \tag{2}$$

$2^* = \frac{2N}{N-2}$ is the critical Sobolev exponent, $\alpha > 0$ is a fixed exponent, and $g \in C^1(\mathbb{R})$ satisfies

$$(\mathrm{H}) \quad \begin{cases} (\mathrm{H})_0 & \lim_{s\to 0} \frac{f(s)}{|s|^{p-2}s} = L_1, \qquad \text{for some } L_1 > 0, \text{ and } p \in \left(2, \frac{2N}{N-2}\right] \\ (\mathrm{H})_\infty & \lim_{s\to\infty} \frac{g(s)}{|s|^{q-2}s} = L_2, \qquad \text{for some } L_2 \geq 0, \text{ and } q \in \left(2, \frac{2N}{N-2}\right) \\ (\mathrm{H})_{g'} & |g'(s)| \leq C(1+|s|^{q-2}), \qquad \text{for } s \in \mathbb{R}. \end{cases}$$

We study the existence of a bifurcated branch of classical positive solutions, containing a turning point, and providing multiplicity of solutions.

This is a joint work with Prof. Rosa Pardo (Universidad Autonoma de Madrid).

Keywords: Slighty subcritical superlinearity, bifurcation results, positive solutions

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