

Implicit and implicit-explicit Lagrange-projection exactly well-balanced finite-volume schemes for the one-dimensional shallow-water system

C. Caballero-Cárdenas¹, M. J. Castro¹, T. Morales de Luna¹, M. L. Muñoz-Ruiz¹

SUMMARY

In this work we present implicit and implicit-explicit first and second order numerical approximations of the shallow-water equations based on a Lagrange-Projection type finite volume strategy. This scheme can be interpreted as a two-step algorithm consisting in first solving the shallow water system in Lagrangian coordinates, which is known as the Lagrangian step, and then projecting the results in Eulerian coordinates, which is known as the Projection step. For the Lagrangian step we propose two different implicit versions: one fully implicit and one implicit-explicit, depending on how the source term is treated. The Projection step will always be done explicitly.

By following this strategy, the acoustic and the transport phenomena can be decoupled and this allows us to design large time step schemes in which the CFL restriction is based on the slower transport waves and not on the acoustic ones. In this work we follow the strategy described in [1, 3] to define the Lagrange-Projection scheme and [2] to ensure its well-balanced character.

Keywords: Lagrange-Projection strategy, IMEX schemes, well-balanced, shallow water equations

AMS Classification: 35Lxx, 65Mxx

References

- [1] M. J. CASTRO, C. CHALONS, AND T. MORALES DE LUNA. A Fully Well-Balanced Lagrange-Projection-Type Scheme for the Shallow-Water Equations. *SIAM Journal on Numerical Analysis* **56**(5), 3071–3098, 2018.
- [2] M. J. CASTRO AND C. PARS. Well-Balanced High-Order Finite Volume Methods for Systems of Balance Laws. *Journal of Scientific Computing* **82**(48), 2020.
- [3] T. MORALES DE LUNA, M. J. CASTRO, AND C. CHALONS. High-order fully well-balanced Lagrange-Projection scheme for shallow water. *Communications in Mathematical Sciences*, **18**(3), 781–807, 2020.

¹Departamento de Análisis Matemático, Estadística e I.O. y Matemática Aplicada
Universidad de Málaga
email: celiacaba@uma.es