Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
Jaca, September 7–9th 2022

Source term linearization when solving blood vessel flow using approximate solvers

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SUMMARY

The flow of blood in elastic vessels such as veins or arteries is difficult to model, specially in the case of the former [1]. It is common to use the Roe solver and its augmented version to solve numerically the hyperbolic conservation laws that govern the flow, in which the solution to the Riemann problem (RP) is given by the eigenvalues of some linearized version of the homogeneous part of the original equations. The inhomogeneous part of the equations accounts for the interactions with the wall, possibly including friction and the stress from sectional variations of the vessel. To deal with them, the Augmented Roe scheme (ARoe) includes the effect the of the source terms over the inner states of the approximate solution given by the homogeneous equations, ensuring equilibrium in well-balanced solutions. However, one disadvantage of the ARoe method is that all the solutions are represented exclusively with shock-type waves, which compromises the resolution of rarefactions, since those can only be represented by a succession of small shock discontinuities. This complication is most prevalent in cases with *transcritical rarefactions*, where the rarefaction fan extends across the sonic point. Harten and Hyman [2] introduced a family of entropy fixes targeted towards solving this defect of the Roe solver based on wave splitting. Indeed, the problematic wave would be separated into two different wave contributions. How to choose these waves gives rise to the different variations of the fix. This work analyzes the application of the wave-splitting method in the case of transcritical rarefactions in venous flow, building up from previous applications to shallow flows and looking at how increasingly complicated methods perform numerically. The goal is to find a compromise between computational cost and precision in order to devise solvers able to simulate blood flows in situations where transcritical rarefactions might occur, such as when liberating a tourniquet.

Keywords: hyperbolic PDEs, transcritical rarefactions, Augmented Roe

AMS Classification: 776H05, 35L04, 76Z05

References

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