Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
Jaca, September 7–9th 2022

Least action solution and least action nodal solution for Schrödinger equation on metric graphs

Colette De Coster[1],

SUMMARY

In this talk, we consider the problem

$$\begin{cases} u'' + |u|^{p-2}u = \lambda u, & \text{on the edges of } \mathcal{G} \\ u \text{ continuous and } \displaystyle\sum_{e \succ \mathrm{V}} \frac{du}{dx_e}(\mathrm{V}) = 0, & \text{at the vertex of } \mathcal{G} \end{cases} \tag{1}$$

set on a metric graph $\mathcal{G}$.

The solutions of this problem are the critical points of the action functional

$$J_\lambda(u) := \frac{1}{2}\|u'\|^2_{L^2(\mathcal{G})} + \frac{\lambda}{2}\|u\|^2_{L^2(\mathcal{G})} - \frac{1}{p}\|u\|^p_{L^p(\mathcal{G})},$$

defined on $H^1(\mathcal{G})$.

Two important levels of J_λ are given by

$$c_\lambda(\mathcal{G}) := \inf_{u \in \mathcal{N}_\lambda(\mathcal{G})} J_\lambda(u)$$

where

$$\mathcal{N}_\lambda(\mathcal{G}) := \{u \in H^1(\mathcal{G}) \mid u \neq 0, \mathrm{d}J_\lambda(u)[u] = 0\}$$

and

$$\sigma_\lambda(\mathcal{G}) := \inf_{u \in \mathcal{S}_\lambda(\mathcal{G})} J_\lambda(u),$$

where $\mathcal{S}_\lambda(\mathcal{G})$ is the set of $H^1(\mathcal{G})$ solutions of the problem (1).

In case $c_\lambda(\mathcal{G})$ is attained, it is well known that the corresponding minimum is a solution of (1). In the first part of this talk we will consider the case where $c_\lambda(\mathcal{G})$ is not attained. We can wonder what are the relations between $c_\lambda(\mathcal{G})$ and $\sigma_\lambda(\mathcal{G})$? Are they equal ? Can we have $c_\lambda(\mathcal{G})$ not attained and $\sigma_\lambda(\mathcal{G})$ attained ?

In the second part of the talk, according to the time left, we will consider the problem of existence of sign-changing solutions of (1).

This is based on joint works with Simone Dovetta (Politecnico di Torino), Damien Galant (UMons - UPHF), Enrico Serra (Politecnico di Torino) and Christophe Troestler (UMons).

[1]Universit Polytechnique Hauts de France
email: colette.decoster@uphf.fr