A TRACING WAVES METHOD FOR THE CONSTRUCTION OF SEISMIC PROPAGATORS

Hélène Barucq, Bertrand Duquet and Frank Prat

Abstract. This work is concerned with a micro-local method for solving the direct problem of acoustic waves scattering in a complex medium. Pseudo-differential theory is used to decompose the scattering problem into a one-way model which accounts for the reflections and transmissions due to the variations of the medium velocity. Then, the multiple reflections can be taken into account and they can be computed separately. This is very interesting in case of imaging processes because the multiple reflections play the role of scrambling the results and a separate computation of multiples is not possible when using finite difference or finite element methods. The numerical solution involves FFTs and the resulting computational burden is lower than the one required by finite difference or finite element techniques.

Keywords: wave equation, pseudo-differential theory,Dirichlet-to-Neumann operator, numerical micro-local analysis

AMS classification: AMS 35S30, 45G80, 58J40, 58J45

§1. Introduction

The numerical solution of a scattering phenomenon may involve different approaches. One of them consists of full-wave methods that are based upon the solution of the exact wave equation by using finite difference or finite element techniques. Any characteristic of the propagation is considered but the related numerical process requires a high computer memory and a high computational burden also [6]. Another solving approach is defined by asymptotic methods. Some of them consists in solving a parabolic approximation of the wave equation [2, 3]. The approximate wave equation is given by a pair of uncoupled equations which describe the wave propagation along an axis. A first paper [2] was concerned with the derivation of the parabolic equations as a high-frequency approximation of the one-dimensional wave equation. This is well-known as the WKB method. Later, Corones [3] improved the modeling by introducing a Bremmer series that allows to account for the reflection and transmission, as opposed to the first modeling [2]. Other asymptotic methods are based upon the tracing ray theory (see [7] and its references). It is also a high-frequency approach which permits a fast computation of the seismograms, as opposed to a full-wave method. However, the accuracy of ray methods decreases in singular regions and high-frequency methods are unsuitable for the wave scattering by strongly heterogeneous objects.

Recently, De Hoop and its co-workers [4, 5] have developed a new method for solving the wave equation. It can be considered as a hybrid method between full-wave and high-frequency methods. The method is based on a system which is derived from the exact wave equation by proceeding to the micro-local analysis of the wave front set in a selected direction which can be the depth for seismic wave modelling but also the normal direction for the scattering by an object. By analogy with the formalism in ray theory, the approach is called the Tracing Wave Methods. Applying the tracing wave theory modifies the wave equation to a system of uncoupled equations which involve the out-going and in-going Dirichlet-to-Neumann operators and coupling terms which account for the reflection and transmission. Then, the new system shows off the main role played by the square-root of the Helmholtz operator. It can be proved that the resulting model generalizes the system proposed in [3]: the parabolic equations coupled with a Bremmer series is an approximation of the tracing wave method in case of small lateral variations of the medium velocity [1]. Besides a lower computational burden as opposed to a full-wave method, the tracing wave method provides a numerical way to separate the multiple reflections which is of great importance for an accurate imaging of the medium by using wave propagation.

In this paper, we intend to apply the micro-local method for the computation of seismograms in complex structures. This provides new and more precise results as compared to [11] where a numerical study was developed in terms of snapshots only. The work is divided into the following parts. We derive the one-way model by using pseudo-differential techniques. Then, we describe the numerical method and focus on its main difficulties. At last, we conclude by a numerical experiment which shows the accuracy of the method as far as the kinematic is concerned but its weak spots also.

§2. Setting of the mathematical model

The propagation of acoustic waves is usually related to the wave equation which can be written as a first order hyperbolic system as follows:

$$\left\{ \begin{array}{c} \nabla p + \partial_t (\rho \mathbf{v}) = \mathbf{f} \\ \partial_t (\kappa\, p) + div\, \mathbf{v} = q. \end{array} \right. \tag{1}$$

The system consists of two coupled equations involving the scalar acoustic pressure p and the particle velocity $\mathbf{v} = {}^t(v_x, v_y, v_z)$. The notation $\mathbf{x} = {}^t(x, y, z)$ defines a generic point in $\mathbb{R}^3$, ∇ is the gradient operator defined for any regular function φ by $\nabla\varphi = {}^t(\partial_x\varphi, \partial_y\varphi, \partial_z\varphi)$ while the divergence div is the scalar operator given by $div\,\mathbf{\Phi} = \partial_x\Phi_x + \partial_y\Phi_y + \partial_z\Phi_z$. As usual, ∂_t denotes the time derivative. The physical parameters are the volume density ρ and the compressibility κ and the velocity of the medium is given by the relation $c(\mathbf{x}) = \sqrt{\rho\kappa}$. In this paper, we assume that ρ is constant. The vector **f** is the volume source density, the scalar q is the volume source density of injection rate and both are data for the system. The propagation medium is given as the half-space $\{z \geq 0\}$ and $z > 0$ describes the underground.

Assume that initial data are given at the surface $\{z = 0\}$. One way to solve system (1) consists in selecting the depth variable z as the preferred direction and to express the motion of the wave along the axis $z > 0$. Such an approach was formerly suggested in [2, 3] but in the simplest case of one-dimensional stratified media which does not require the use of pseudo-differential techniques. The complexity of the propagation medium prohibits from using the same arguments than in the previous papers and the motion in the z-direction can be written only by using the formalism of pseudo-differential operators. To transform (1) requires first to eliminate the time derivative. This is achieved easily by using a Fourier-Laplace transform with dual variable ω. Then system (1) modifies to:

$$\begin{cases} \nabla_t \widehat{p} + i\omega\left(\rho\widehat{\mathbf{v}}\right)' = \left(\widehat{\mathbf{f}}\right)', \\ \partial_z\widehat{p} + i\omega\left(\rho\widehat{v_z}\right) = \widehat{\mathbf{f}}_z, \\ i\omega\kappa\,\widehat{p} + div\ \left(\widehat{\mathbf{v}}\right)' + \partial_z\widehat{v}_z = \widehat{q}, \end{cases}$$

where $\nabla_t \widehat{p} =^t (\partial_x\widehat{p}, \partial_y\widehat{p})$, $\widehat{\mathbf{v}}' =^t (\widehat{v}_x, \widehat{v}_y)$ and $\widehat{\mathbf{f}}' =^t \left(\widehat{f}_x, \widehat{f}_y\right)$. As usual, $\widehat{\varphi}$ denotes the Fourier transform of φ. Here, it is the Fourier transform with respect to the time $t > 0$, the causality principle being satisfied by the wave solution. The first equation allows to eliminate the tangential unknown $\widehat{\mathbf{v}}' =^t (\widehat{v}_x, \widehat{v}_y)$ in the third equation and we get a system of the form:

$$\left(\mathbf{D}_z + L\right)\mathbf{U} = \mathbf{F} \tag{2}$$

where $\mathbf{U} =^t (\widehat{p}, \widehat{v_z})$, $\mathbf{D}_z = \partial_z \mathbb{I}_2$ and $\mathbb{I}_2$ denotes the identity. The operator L is defined by:

$$L = \begin{pmatrix} 0 & i\omega\rho \\ i\omega\kappa + \frac{i}{\omega\rho} div \nabla_t & 0 \end{pmatrix}$$

and the source $\mathbf{F}$ is given by $\mathbf{F} =^t \left(\widehat{\mathbf{f}}_z, \widehat{q} - \dfrac{1}{i\omega\rho} div \left(\widehat{\mathbf{f}}\right)'\right)$. Factorizing by $i\omega$, operator L can be written as $L = i\omega L^\sharp$ where $L^\sharp = L^\sharp\left(\mathbf{x}', \dfrac{1}{\omega}\partial_{\mathbf{x}'}\right)$. According to the theory developed by Hörmander [8], the operator $L^\sharp$ is a pseudo-differential operator in OPS^1 depending on the parameter $1/\omega$ and if $\mathcal{L}^\sharp$ denotes its symbol, we have the representation: for any test-function φ,

$$\left\langle L^\sharp, \varphi \right\rangle = \frac{1}{(2\pi)^2} \int\int \mathcal{L}^\sharp\left(\mathbf{x}', \frac{\mathbf{k}'}{\omega}\right) \varphi\left(\mathbf{s}'\right) e^{-i(\mathbf{x}'-\mathbf{s}').k'} d\mathbf{s}' d\mathbf{k}'$$

where $\mathbf{s}' =^t (s_x, s_y) \in \mathbb{R}^2$ and $\mathbf{k}' =^t (k_x, k_y)$ is the dual variable to $\mathbf{x}'$ such that the symbol of ∇_t is $i\mathbf{k}'$. The symbol $\mathcal{L}^\sharp$ is then defined by:

$$\mathcal{L}^\sharp = \begin{pmatrix} 0 & \rho \\ \kappa - \frac{|\mathbf{k}'|^2}{\omega^2\rho} & 0 \end{pmatrix},$$

with $|\mathbf{k}'|^2 = k_x^2 + k_y^2$.

The solution of (2) gives a description of the motion of the wave $\mathbf{U}$ along the depth z. The formalism of pseudo-differential operators allows to extend a classical approach for solving differential systems with constant coefficients which consists in making diagonal the matrix L. In case of pseudo-differential operators (or differential operators with variable coefficients), Taylor [12] has developed a diagonalization process for strictly hyperbolic systems and herein, we follow his approach.

According to system (2), L describes the variations of $\mathbf{U}$ along the axis z. Since L is a strictly first-order hyperbolic operator, its symbol $\mathcal{L}$ (which equates the principal symbol) admits two single eigenvalues. One describes the behavior of the wave front set along the down-going bicharacteristic while the other is related to the up-going bicharacteristic. With regard to the propagation, making $\mathcal{L}$ allows to decompose $\mathbf{U}$ into a downward and a upward part. The eigenvalues of $\mathcal{L}^\sharp$ are given by $\pm\gamma$ with $\gamma = \left(\dfrac{1}{c^2(\mathbf{x})} - \dfrac{|\mathbf{k}'|^2}{\omega^2}\right)^{1/2}$. The frequency $\mathbf{k}'$ belongs to $\mathbb{R}^2$ which is divided into three regions. The first region is $\left\{|\mathbf{k}'|^2 < \dfrac{\omega^2}{c^2(\mathbf{x})}\right\}$ where the eigenvalues of $\mathcal{L}$ are imaginary. It is usually called the hyperbolic region and it corresponds to the propagation modes. In that case, we will call γ the down-going eigenvalue while $-\gamma$ is the up-going one. The second region is $\left\{|\mathbf{k}'|^2 > \dfrac{\omega^2}{c^2(\mathbf{x})}\right\}$, the so-called elliptic region. In this frequency zone, the expression of the eigenvalues involves the principal determination of the square-root in the complex plane and then, the eigenvalues of $\mathcal{L}$ are real. The eigenvalue γ is now related to an evanescent mode in this region while the other eigenvalue $-\gamma$ is rather associated to a blowing mode (increasing as ω increases). In this region, no propagation phenomenon occurs. At last, the region $\left\{|\mathbf{k}'|^2 = \dfrac{\omega^2}{c^2(\mathbf{x})}\right\}$ is the region of grazing rays in which the problem degenerates into a double eigenvalue one and the system is no more strictly hyperbolic. As in the previous case, there is no propagation of waves.

Next, we can define a diagonal operator Λ which is a pseudo-differential operator in OPS^1 whose symbol is the matrix $\begin{pmatrix} \gamma & 0 \\ 0 & -\gamma \end{pmatrix}$. For any fixed z, this operator is the Dirichlet-to-Neumann operator which is frequently associated to the wave equation in the context of scattering problems. Operator Λ and operator $L^\sharp$ are linked by the relation $L^\sharp = P\Lambda P^{-1}$ where P stands for the pseudo-differential operator with symbol equal to the matrix $\mathcal{P}$ constructed from the eigenvectors of $\mathcal{L}^\sharp$. It can be chosen in the form:

$$\mathcal{P} = \begin{pmatrix} \rho & \rho \\ -i(\gamma/\omega) & i(\gamma/\omega) \end{pmatrix}$$

and then, $P \in \mathrm{OPS}^0$. Its inverse P^{-1} exists if $\mathcal{P}^{-1}$ exists and it is a pseudo-differential operator of order 0 also whose principal symbol is $\mathcal{P}^{-1}$. Then, the previous decomposition of L onto the eigenvectors basis allows to derive an auxiliary model, the so-called one-way model which is developed in the next section.

§3. The one-way model

The diagonalization of L provides a direct way to describe the down-going and the up-going motions of the acoustic wave $\mathbf{U}$. This is achieved by changing the unknown such that (2) is re-written as a diagonal system on the left and coupled terms on the right which describe the reflection and transmission events. The new unknown $\mathbf{V}$ is expressed from $\mathbf{U}$ by the relation $\mathbf{V} = P\mathbf{U}$ which shows off $\mathbf{V}$ is the projection of $\mathbf{U}$ onto the generalized eigenspaces of L. That means, according to the definition of Λ, the first component of $\mathbf{V}$ represents the downward part of $\mathbf{U}$ along the axis $z > 0$ while the second component of $\mathbf{V}$ is the upward part of the wave field. By plugging $\mathbf{V}$ into (2), we get the new system :

$$(\mathbf{D}_z + i\omega\Lambda)\,\mathbf{V} = -P^{-1}\left(\partial_z P\right)\mathbf{V} + P^{-1}\mathbf{F}. \tag{3}$$

The left side of (3) consists of two uncoupled equations, the so-called one-way system which involves the diagonal operator Λ only. The right side is composed of the data and of a coupling term which only depends on P and describes the reflection and the transmission.

Remark. We can observe that if P does not depend on z, the coupling term vanishes and (3) generalizes the WKBJ model that was used in [2] in case of homogeneous one-dimensional stratified media. Moreover, since we have chosen $\mathcal{P}$ such that $P \in OPS^0$, the changing of unknown $\mathbf{V} = P\mathbf{U}$ preserves the regularity of $\mathbf{U}$.

To define the inverse of P, we have to compute its symbol. Note that according to the theory of pseudo-differential operators, P^{-1} is well-defined because $\mathcal{P}^{-1}$ exists with:

$$\mathcal{P}^{-1} = \frac{1}{2}\begin{pmatrix} \rho^{-1} & -i\omega/\gamma \\ \rho^{-1} & i\omega/\gamma \end{pmatrix}.$$

Then, we propose to replace P^{-1} by the operator whose symbol is $\mathcal{P}^{-1}$ exactly. This amounts to approximate the symbol of P^{-1} by its principal symbol. In the following, we keep the notation P^{-1} but we do not consider the exact operator P^{-1}. The reason is that the exact symbol of P^{-1} is a series of matrices. The terms of any matrix are homogeneous symbols that are combinations of the $x^{'}$ and $k^{'}$-derivatives of γ^{-1}. Hence, the exact symbol is not easy to compute and we prefer to consider an approximation.

As far as the coupling terms are concerned, it is convenient to re-write them like:

$$-P^{-1}\left(\partial_z P\right) = \begin{pmatrix} T & R \\ R & T \end{pmatrix}$$

where T denotes the transmission operator and R is the reflection one. Then, T and R have symbols $\mathcal{T}$ and $\mathcal{R}$ which are signs unlike with $\mathcal{R} = \frac{1}{2}\gamma^{-1}\partial_z\gamma$.

§4. The down-going operator

In this section, we focus on the determination of the down-going eigenvalue and its inverse since both these symbols play a main role in the computational method. Indeed, the main term in the symbol of P is given by γ and since we have chosen to approximate the symbol of P^{-1} by its principal symbol, we only have to determine γ^{-1}. In order to limit the difficulty of implementation and the computational costs, it is convenient to approximate the square-root γ by some polynomial. In the context of wave propagation problems, it is quite usual to introduce a background medium related to a velocity which only depends on the depth z. For any fixed z, it can be defined by:

$$c_0\left(z\right) = \min_{\mathbf{x}^{'}} c\left(\mathbf{x}^{'}, z\right).$$

Then, the wave equation set in the fictitious medium with velocity $c_0(z)$ is related to the eigenvalue $\gamma_0\left(z,\frac{\mathbf{k}'}{\omega}\right)=\left(\frac{1}{c_0^2(z)}-\frac{|\mathbf{k}'|^2}{\omega^2}\right)^{1/2}$ and we can re-write the down-going eigenvalue as:

$$\gamma\left(\mathbf{x}',z,\frac{\mathbf{k}'}{\omega}\right)=\gamma_0\left(z,\frac{\mathbf{k}'}{\omega}\right)\sqrt{1-\frac{\delta\left(\mathbf{x}',z\right)}{\left(\gamma_0\left(z,\frac{\mathbf{k}'}{\omega}\right)\right)^2}} \tag{4}$$

where $\delta\left(\mathbf{x}',z\right)=\frac{1}{c_0^2(z)}-\frac{1}{c^2\left(\mathbf{x}',z\right)}$ is a parameter which accounts for the lateral variations of velocity. The reference velocity has been chosen such that $\frac{\delta\left(\mathbf{x}',z\right)}{\left(\gamma_0\left(z,\frac{\mathbf{k}'}{\omega}\right)\right)^2}$ is smaller than 1 and then, the square root in (4) is approximated by the truncated Taylor expansion:

$$\gamma\left(\mathbf{x}',z,\frac{\mathbf{k}'}{\omega}\right)\simeq\gamma_0\left(z,\frac{\mathbf{k}'}{\omega}\right)\left(1+\sum_{j=1}^{m}a_j\frac{\delta^j\left(\mathbf{x}',z\right)}{\left(\gamma_0\left(z,\frac{\mathbf{k}'}{\omega}\right)\right)^{2j}}\right). \tag{5}$$

Formula (5) is of practical value quite easily since the variables $\mathbf{x}'$ and $\mathbf{k}'$ are separated. Hence, as far as the numerical implementation of (5) is concerned, the storages of $\mathbf{x}'$ and $\mathbf{k}'$ are uncoupled which minimizes the memory storage and the related computational burden of the method is improved.

As far as the integration of γ^{-1} is concerned, a difficulty arises from the fact that $\frac{1}{c^2(\mathbf{x})}-\frac{|\mathbf{k}'|^2}{\omega^2}$ can vanish and, even if the inverse of the square-root is integrable near the origin, there is a singularity from a numerical point of view. This is why we use a numerical artefact which consists in avoiding the origin by integrating along a half-circle centered at $\frac{1}{c^2(\mathbf{x})}$ and with radius R. The numerical method is very sensitive to the choice of R. We refer to [9] for a detailed discussion on the fitting of R. In this paper, we do not develop this question but we illustrate it by numerical results.

§5. The propagators

The propagator $G=\begin{pmatrix} G^+ & 0 \\ 0 & G^- \end{pmatrix}$ is the diagonal operator defined by the pair $G^\pm$ which are the inverse of $(\mathbf{D}_z\pm\Gamma)$, providing Γ is the operator with symbol γ. Operator G^+ describes just the propagation from the top of the medium to its bottom while G^- is related to the retro-propagation. Both these operator are involved for solving the one-way model as follows. Assuming that $G^\pm$ are well-defined, (3) modifies to

$$\left(\mathbb{I}+GP^{-1}\left(\partial_z P\right)\right)\mathbf{V}=GP^{-1}\mathbf{F}, \tag{6}$$

where $\mathbf{I}$ stands for the identity in $\mathbb{C}^2$. The unknown V is then obtained by inverting $\mathbb{I} + GP^{-1}(\partial_z P)$. This can be achieved because the solution operator is a perturbation of the identity $I \in OPS^0$ by an operator in OPS^{-1}. Moreover, the theory of pseudo-differential operators ensures that the inverse operator is a series of the form:

$$\left(\mathbb{I} + GP^{-1}(\partial_z P)\right)^{-1} = \mathbb{I} - GP^{-1}(\partial_z P) + \left(GP^{-1}(\partial_z P)\right)^2 - \ldots + (-1)^m \left(GP^{-1}(\partial_z P)\right)^m + \ldots$$

modulo a regularizing operator. This is what we call the Bremmer series, just as was formerly introduced by Corones in [3]. From a numerical point of view, we use a truncated expansion. The first term of the series (the zero-order one) is related to the source propagation. The next term (the first-order one) describes the primary reflection and the (m+1)th gives the (m-1)th multiple, $m \geq 2$.

According to the semi-group theory, $G^{\pm}$ is characterized by its kernel $K^{\pm}$ and the Trotter formula (see [10]). From a numerical point of view, we proceed to an approximation of the kernel of $G^{\pm}$ which is based upon a discretization of the medium into twice sections of depth Δz and the fist-order approximation is given by:

$$K^{\pm}\left(\mathbf{x}', z, \mathbf{s}', s_z\right) = \frac{1}{(2\pi)^2} \int e^{i\omega\left(\mathbf{x}' - \mathbf{s}'\right).\mathbf{k}'} \exp\left(\pm i\omega\gamma\left(\mathbf{x}', \frac{z+s_z}{2}, \frac{\mathbf{k}'}{\omega}\right)\Delta z\right) d\mathbf{k}'.$$

In order to speed up the integration of the kernel, whence the computation of the propagators, we proceed to the approximation of $\exp\left(\pm i\omega\gamma\left(\mathbf{x}', \frac{z+s_z}{2}, \frac{\mathbf{k}'}{\omega}\right)\Delta z\right)$, providing Δz is small. But we have to correct the amplitude of the approximation since the exact symbol belongs to the unit circle. Hence,

$$\exp\left(\pm\gamma\left(\mathbf{x}', \frac{z+s_z}{2}, \frac{\mathbf{k}'}{\omega}\right)\Delta z\right) \simeq N\left(\mathbf{x}', \frac{z+s_z}{2}, \frac{\mathbf{k}'}{\omega}\right)\left(1 \pm i\omega\gamma\left(\mathbf{x}', \frac{z+s_z}{2}, \frac{\mathbf{k}'}{\omega}\right)\Delta z\right)$$

where $N\left(\mathbf{x}', \frac{z+s_z}{2}, \frac{\mathbf{k}'}{\omega}\right)$ is a normalization factor. Herein, if $w = 1 + p + iq$, $(p,q) \in (\mathbb{R})^2$,

$$N(w) = \exp(iq)\left|1 + \frac{p}{1+iq}\right|^{-1}\left(1 + \frac{p}{1+iq}\right)$$

where $|\cdot|$ denotes the modulus in the complex plane. This amounts to approximate $1 + iq$ by $\exp(iq)$ and to project the complex number on the unit circle dividing by the modulus of $1 + \dfrac{p}{1+iq}$.

§6. Numerical experiments

We illustrate the method by some two-dimensional tests. We consider the simplest case of three homogeneous media separated by plane interfaces($\delta = 0$). The computational domain is 4.4 kilometers wide and 3 kilometers deep. The source point is located at $x = 2200$ meters. The number of discretization points is equal to 440 and the receivers are located at$x_j = j\Delta x$. The source is a Gaussian. We use the finite difference software Twist ++ developed at the Geoscience Research Center (Total, London) and the results are compared in a computational

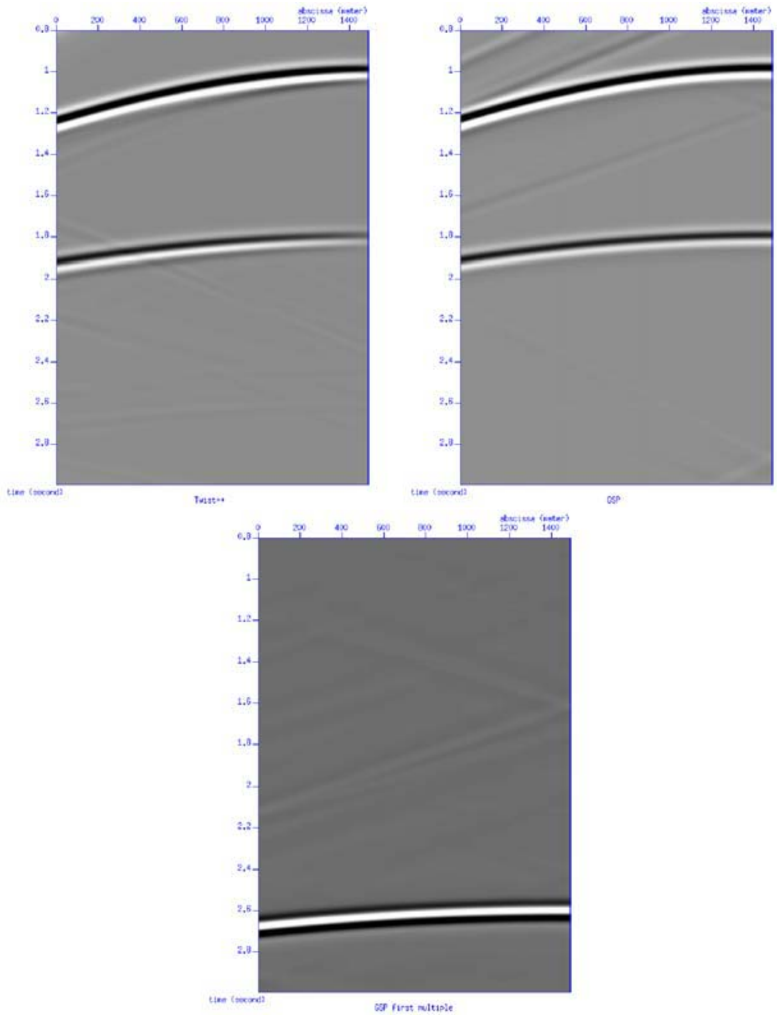

Figure 1: *On the top left figure, Twist++ result; on the top right, GSP result; on the bottom figure, the first multiple alone with GSP*

window which is 1.5 kilometers wide. The ordinate gives the values of the time arrivals. On Fig.1, the first left pattern represents the results obtained with Twist++ while the right pattern is related to the tracing wave method. As far the kinematic is concerned, the results are in good agreement. The third pattern at Fig.1 shows the first multiple which can be computed separately by the tracing wave approach only.
The second test is based upon a velocity model (left Fig.2) which is related to a medium divided into two regions by an interface with a slope which allows to account for the lateral variations of the velocity ($\delta \neq 0$). The computational domain is a square with side equal to 4 kilometers. The point source is located at $x = 2000$ meters and is equal to the second derivative of a time gaussian centered at the origin. The number of discretization points along the horizontal axis is equal to 400 and the receivers are located at each node $x_j = j\Delta x.$

For this model, the seismic response can be computed analytically. As far as the kinematic is concerned, the results are correct. Nevertheless, some spurious events occur. On the top of the computational domain, we can observe some noisy events which interfere in the pattern like

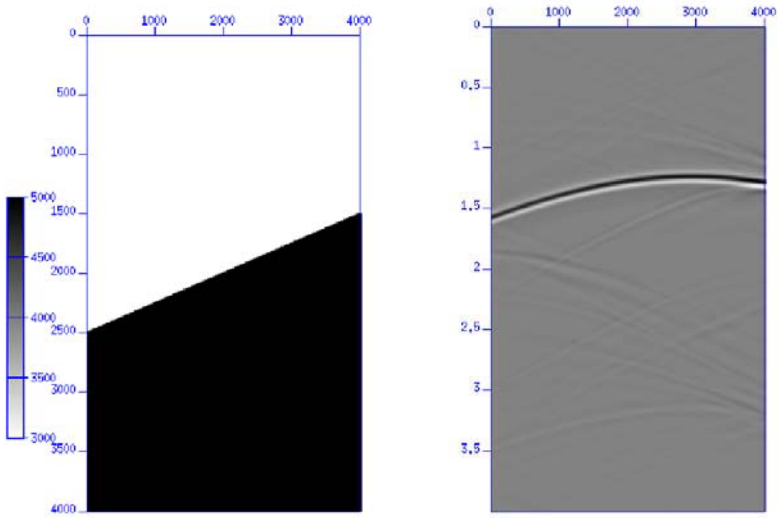

Figure 2: *On the left figure, the velocity model as a function of* (x_1, x_3) *, on the right figure, the arrival time (Y-axis) at the receivers (X-axis)*

crosspieces. Their origin can be multiple. Indeed, we use a collection of FFTs which make the signals periodic and create artificial sources. Moreover, the treatment of the singularity in the grazing zone by a contour in the complex plane seems to cause artificial noise. Indeed, beyond avoiding the singularity for γ^{-1}, the frequencies go from the hyperbolic region to a region, the so-called elliptic zone, which does not play a role for the propagation. Hence, in order to eliminate the elliptic frequencies, we multiply the related solution by a decreasing exponential and this can generate some noise into the numerical result. This numerical artefact is related to the value of the reference velocity and then on the value of the contour radius R. The best situation will be given when the reference velocity is quite close to the velocity. But this is not an interesting situation since we intend to use the method when the lateral variations of the velocity are high.

Actually, some ideas have been explored for the improvement of the numerical method. The first consists in optimizing the computation of the reflection operator and it will be the subject of a further work. The second deals with the computation of γ^{-1} and the use of the contour for avoiding the grazing zone. At last, some investigations concern the normalization in the propagator. Some numerical tests seem to indicate that it could generate some instabilities.

Acknowledgements

The authors thank Total for its financial support and Henri Calandra who initiated this research subject.

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Hélène Barucq and Frank Prat
Laboratoire de Mathématiques Appliquées, FRE 2570
Université de Pau et des Pays de l'Adour
IPRA, BP1155, 64013 Pau Cedex, France
helene.barucq@univ-pau.fr

Bertrand Duquet
IFP, currently seconded to Total
Total CSTJF
64000 Pau FRANCE