Vehicle routing problems with soft time windows: An optimization based approach

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Abstract. The classical vehicle routing problem involves to design a set of routes for a fleet of vehicles based at one central depot that is required to service a number of geographically dispersed customers, while minimizing the total travel distance or the total distribution cost. Each route originates and terminates at the central depot and customers' demand are known.

In many practical distribution problems, a time window is associated to each customer, defining a time interval in which the customer should be supplied. This is the vehicle routing problem with time windows. This works investigates the use of Goal Programming to modeling a vehicle routing problem with soft delivery time window constraints. In this problem vehicles are allowed to service customers before and after the earliest and latest time windows bounds. This relaxation comes at the expense of appropriate penalties that reflect the effect that time windows violations have on the customers' satisfaction.

§1. Introduction

In such a competitive world as the current one, it is evident that companies should make operational and strategic decisions in order to optimize and to manage the processes involved in their supply chains more efficiently. Amongst them, the delivery of commodities at low cost, with high quality of service and with short delay times. This warranties not only a good service to customers but a saving in warehousing and distribution costs. The problem of physical distribution of goods to customers' locations is of particular importance since it account for a large proportion of the overall operational costs of a producer. Hence, effective and efficient management of transportation and distribution of goods is becoming increasingly important both from the point of view of theoretical research and from the point of view of practical applications.

Regarding these planning operations, three different decision problems can be identified. First one, to cluster customers in geographically areas. Otherwise, the complexity of the resulting model, in terms of number of variables and constraints, severely affects the efficiency of

the algorithms that can be employed in practical situations to generate and evaluate feasible solutions and, eventually, obtain an optimal o close to optimal solution. Second decision problem is to design, over a given planning horizon, an optimal schedule to satisfy customers' demand. Finally, third decision problem is to determine an optimal set of routes to supply from a central depot to the customers in its geographical area. This is the problem specifically considered in this work.

These problems are generically known as Vehicle Routing Problems (VRP). The classical VRP consists in determining the best set of routes for a fleet of vehicles based at a single central depot to distribute goods to a set of customers geographically dispersed, while minimizing the total travel distance or the total distribution cost. Due to its importance, there is a vast literature that addresses modeling and solution aspects of the VRP. Every paper presents a view of the problem, considering different features of the system and a different approach to solving the problem (see [2, 3, 6, 7, 16] and references therein).

When, in addition, bounds exist on the moments of the day in which deliveries should take place these problems are known as Vehicle Routing Problems with Time Windows (VRPTW). In these problems every customer has a time interval (time window) associated wherein he/she should be supplied. These constraints are hard constraints when a route is not feasible if the service of a customer either starts before the earliest time or ends after the latest time of the day established by the time window ([4, 5, 8, 9, 11, 14]). In other cases, both lower and upper bounds of the time window need not be satisfied, but can be violated at a penalty. These are Vehicle Routing Problems with Soft Time Windows (VRPSTW) [1, 10, 12, 15].

All these combinatorial optimization problems have been proven to be NP-hard and only relatively small instances can be solved to optimality. Methodology involves the use of branch and bound methods, Lagrangian relaxation, etc. For bigger problems, researchers usually focus on heuristic and meta-heuristic (tabu search, genetic algorithms, simulated annealing) methods to derive approximate solutions of acceptable quality in reasonable computational time. In general, heuristic methods construct routes step-by-step, giving criteria to identify the following customer to be added to the route under construction and the insertion point.

In this work we propose to model the WRPSTW as a mixed-integer goal programming problem. In the objective function, penalties are assigned to deviational variables which reflect the violation of time windows. These penalties weight the effect of not satisfying the customers' preferences on the interval time during which the delivery should have taken place. This approach extends the model proposed by [12], since the number of vehicles is not required to be one, but is a variable which is determined in the optimization process. Moreover, service can either start before the earliest time or ends after the latest time of the day fixed by the customer. This optimization model is useful when planning daily distribution from a central depot to a small number of customers. In addition, this model allows us to evaluate the consequences of assigning different penalties to the violation of time windows depending on the customer.

The remainder of the paper is organized as follows. We start in Section 2 by presenting a comprehensive description of the problem and formulating the goal programming model. In Section 3 a small instance is presented in which there is a single central depot supplying eleven customers. Based on this example, we show the difficulties (in terms of computational time involved) of getting an exact optimal solution of the model. Finally, some ideas for future work are given.

§2. Model formulation

As previously mentioned we consider the problem of delivering a large number of goods, very different in weight and volume, from a single central depot to a number of geographically dispersed customers. These customers pose demands for goods with a specified frequency and act as sales points for final customers. We assume that customers' demand is known when delivery routes are established. Although goods can be evaluated in terms of weight, volume or other characteristics of interest, in the proposed model only weight is relevant. The travel time between the central depot and each customer's location, as well as between each pair of customer's locations is also known.

Let $\mathcal{G} = [\mathcal{N}, \mathcal{A}]$ be a directed network associated to the system, where $\mathcal{N} = \{1, \ldots, n\}$ is the set of nodes (each representing the central depot or a customer's location) and $\mathcal{A} = \{(i, j) : i, j \in \mathcal{N}\}$ is the set of directed arcs (each representing a direct connection). Index 1 refers to the central depot, while indices 2 to n refer to the customers.

Let c_{ij} denote the cost and t_{ij} denote the travel time associated with going from node i to node j through arc (i, j).

Each customer i poses a demand q_i and requires a service time s_i. Notice that, in general, service time is a function of demand $s_i = s_i(q_i)$. Moreover, each customer i has established an interval $[e_i, l_i]$ which indicates his/her preferences regarding the moment of the day in which he/she should be supplied. The lower bound e_i indicates the earliest time in which the service of customer i should start. Similarly, the upper bound l_i indicates the latest time in which the service of customer i should finish. Besides, we assume that deliveries cannot be split up amongst vehicles, that is to say, each customer is served by a single vehicle.

To deliver goods there exists a fleet of V vehicles with known capacity. Notice that V only indicates the maximum number of vehicles that can be used for deliveries. The actual number of vehicles that will be used is one of the variables of the model. Vehicles are initially located at the central depot. Moreover, the route of each vehicle starts and finishes at the central depot. Let C_k denote the capacity of the vehicle k and w_k the fixed cost associated with actually using the vehicle, $k = 1, \ldots, V$.

The purpose of the model is to determine the number of vehicles which should be used to supply the customers and to design the set of vehicle routes to minimize operational costs, while meeting the preferences of customers regarding the time of the day in which they should be supplied. The goals of the model are:

(1) minimize total cost, and

(2) satisfy time window preferences of customer i, $i = 2, \ldots, n$.

To formulate the model we define the following variables:

$$x_{ij}^k = \begin{cases} 1, & \text{if vehicle } k \text{ travels directly from node } i \text{ to node } j; \\ 0, & \text{otherwise.} \end{cases}$$

$$z_k = \begin{cases} 1, & \text{if vehicle } k \text{ is actually used;} \\ 0, & \text{otherwise.} \end{cases}$$

a_i = arrival time (service starting) to customer i.

p_i = departure time (service finishing) from customer i.

p_1^k = departure time from the central depot of vehicle k.

We assume that delivery starts as soon as the vehicle arrives to the customer and that the vehicle leaves the customer as soon as the delivery has been completed. Hence

$$p_i = a_i + s_i, \quad i = 2, \ldots, n$$

Moreover, in the goal programming model, $g_i^+, \tilde{g}_i^+$ and $\hat{g}^+$ are deviational variables representing the amount by which we numerically exceed the corresponding goal and $g_i^-, \tilde{g}_i^-$ are deviational variables representing the amount by which we are numerically under the corresponding goal. Finally, $\hat{H}$, H_i y $\tilde{H}_i$ denote penalties per unit of deviation from each goal and Z stands for a lower bound on the total operational cost of delivery.

Given the above defined variables, the problem can be formulated as follows:

$$\min \quad \hat{H}\hat{g}^+ + \sum_{i=2}^n H_i g_i^- + \sum_{i=2}^n \tilde{H}_i \tilde{g}_i^+$$

subject to

$$\sum_{i=1}^n \sum_{k=1}^V x_{ij}^k = 1 \qquad j = 2, \ldots, n \tag{1}$$

$$\sum_{j=1}^n \sum_{k=1}^V x_{ij}^k = 1 \qquad i = 2, \ldots, n \tag{2}$$

$$x_{ij}^k - z_k \leq 0 \qquad i, j = 1, \ldots, n \tag{3}$$

$$\sum_{j=2}^n x_{1j}^k \leq 1 \qquad k = 1, \ldots, V \tag{4}$$

$$\sum_{i=2}^n x_{i1}^k \leq 1 \qquad k = 1, \ldots, V \tag{5}$$

$$\sum_{i=1}^n x_{ir}^k - \sum_{j=1}^n x_{rj}^k = 0 \qquad r = 1, \ldots, n; \ k = 1, \ldots, V \tag{6}$$

$$\sum_{i=2}^{n} q_i \sum_{j=1}^{n} x_{ij}^k \le C_k \qquad k = 1, \ldots, V \tag{7}$$

$$p_j - p_1^k + (1 - x_{1j}^k)M \ge s_j + t_{1j} \qquad j = 2, \ldots, n; \ k = 1, \ldots, V \tag{8}$$

$$p_j - p_1^k - (1 - x_{1j}^k)M \le s_j + t_{1j} \qquad j = 2, \ldots, n; \ k = 1, \ldots, V \tag{9}$$

$$p_j - p_i + (1 - x_{ij}^k)M \ge s_j + t_{ij} \qquad i, j = 2, \ldots, n; \ k = 1, \ldots, V \tag{10}$$

$$p_j - p_i - (1 - x_{ij}^k)M \le s_j + t_{ij} \qquad i, j = 2, \ldots, n; \ k = 1, \ldots, V \tag{11}$$

$$p_i + g_i^- - g_i^+ = e_i + s_i \qquad i = 2, \ldots, n \tag{12}$$

$$p_i + \tilde{g}_i^- - \tilde{g}_i^+ = l_i \qquad i = 2, \ldots, n \tag{13}$$

$$\sum_{k=1}^{V} \sum_{(i,j) \in \mathcal{A}} c_{ij} x_{ij}^k + \sum_{k=1}^{V} w_k z_k - \hat{g}^+ = Z \tag{14}$$

$$x_{ij}^k = 0, 1, \ z_k = 0, 1 \qquad i, j = 1, \ldots, n; \ k = 1, \ldots, V$$

$$p_1^k, \ p_i, \ g_i^-, \ g_i^+, \ \tilde{g}_i^-, \ \tilde{g}_i^+, \ \hat{g}^+ \ge 0, \qquad i = 2, \ldots, n; \ k = 1, \ldots, V$$

Constraint (1) ensures that only a vehicle arrives to customer j. Constraint (2) ensures that only a vehicle leaves customer i. Constraint (3) ensures that all used vehicles are payed for. Constraint (4) imposes that all vehicles leave from the central depot. Constraint (5) imposes that all vehicles return to the central depot. Constraint (6) is the typical flow conservation equation that ensures the continuity of each vehicle route. Constraint (7) ensures that the total load allocated to vehicle k does not exceed its capacity.

Constraints (8), (9), (10) and (11) guarantee the feasibility of the schedule for each vehicle. Constraints (12) and (13) are goal programming constraints on time window preferences of customers. Constraint (14) allows us to assign a penalty to a deviation from a targeted total delivery cost Z. In the previous setting it is implied that variables $x_{ii}^k, i = 1, \ldots, n$ are not considered or, equivalently, they are equal to zero.

The proposed model for the VRPSTW is a mixed-integer goal programming problem with 0-1 variables, which can be solved by applying usual techniques [13]. It is worth noting that computational time needed to exactly solve this kind of problems strongly depends on the number of 0-1 variables involved.

When all vehicles are identical the model can be simplified, since it is not necessary to identify the kind of vehicle which is going from the customer i to the customer j through the connection (i, j). Hence, the number of 0-1 variables dramatically decreases since it is enough to define a single 0-1 variable x_{ij} associated with each arc (i, j). This reduction makes it possible to solve the model with a larger number of customers in a reasonable amount of computational time.

Table 1: Customer data

	C 2	C 3	C 4	C 5	C 6	C 7	C 8	C 9	C 10	C 10	C 11
q_i	700	420	520	200	160	600	550	750	670	1180	650
s_i	35	21	26	10	8	30	28	37	34	60	33
e_i	–	7 h.	15 h.	15 h.	7 h.	–	7 h.	7 h.	–	15 h.	15 h.
l_1	–	8 h.	16 h.	16 h.	8 h.	–	8 h.	8 h.	–	16 h.	16 h.

Table 2: Travel times t_{ij}

	C D	C 2	C 3	C 4	C 5	C 6	C 7	C 8	C 9	C 10	C 10	C 11
C D	0											
C 1	70	0										
C 2	49	82	0									
C 3	90	130	51	0								
C 4	91	47	48	90	0							
C 5	76	116	37	17	76	0						
C 6	73	59	110	155	107	141	0					
C 7	74	135	114	156	106	142	85	0				
C 8	85	143	125	167	167	152	85	10	0			
C 9	117	157	157	198	205	184	99	42	35	0		
C 10	50	107	86	128	128	113	114	46	56	58	0	
C 11	73	42	94	143	90	128	20	42	104	118	110	0

§3. Illustrative case

We consider the problem of delivering goods from a central depot to eleven customers geographically dispersed, i.e. the network has twelve nodes (see the geographical distribution, for instance, in figure 1). Products are loaded on appropriate vehicles at the central depot and, afterwards, they are transported via a road network to the customers' locations. At each location, goods demanded by the customer are unloaded from the vehicle, and then vehicles travel to the next customers' location where the process is repeated. After all deliveries have been performed, vehicles return to the central depot.

The model's input data (demands, service times and time windows) are displayed in table 1. The symbol '-' stands for the no existence of time window. Travel times t_{ij} are given in table 2. The fleet of vehicles is homogeneous. For each vehicle, the capacity is 3000 and the fixed cost is 100. Moreover, $c_{ij} = 0.2\ t_{ij}$. Notice that at least three vehicles are needed to perform the deliveries since the total demand amounts to 6400.

The model was solved by using Lingo 8.0 and run on a PC Pentium 4, 1.4GHz, 512MB under Windows XP. The bound Z was obtained by solving a classical VRP, that is to say, all constraints related to the moment of the day in which delivery should take place have been eliminated. In addition, the objective function minimize the total operational cost (routes' cost plus vehicles' cost). This problem has 145 variables, from which 133 are binary variables. It

took 5 seconds to get the optimal solution. Total cost is 449 and three vehicles are used. Routes are shown in figure 1-(a).

When time windows are hard, the model has 157 variables, from which, as above, 133 are binary variables. It took 17 seconds to get the optimal solution. Total cost is 916 and six vehicles are used. Routes are displayed in figure 1-(b). Notice that in this case total cost is more than twice the cost when there are not time window constraints involved.

To solve the goal programming model proposed in the paper, we chose the following penalties: $\hat{H} = 1$ and $H_i = \tilde{H}_i = 10, \ i = 2, \ldots, 12$. This problem has 206 variables, from which 133 are binary variables. It took 5 minutes to get the optimal solution. Five vehicles are used. The routes are shown in figure 2-(a). Every customer is supplied according to his/her preferences, i.e. within his/her time window, except for customer 3 whose delivery starts six minutes before the earliest time preferred by this customer. Total cost is 798,4. Notice that we save a 13% of the cost by allowing that the delivery to customer 3 starts six minutes early.

If, in addition, we impose that at most four vehicles are used, it took 7 minutes 3 seconds to get the optimal solution. Routes are displayed in figure 2-(b). In this case, the delivery of customer 3 starts two minutes before the earliest time and the deliveries of customers 6 and 9 end four and forty six minutes after the latest time, respectively. Total cost is 657,6, that is to say, 28% less.

It is worth pointing out that penalties in the objective function affect the computational time needed to get an optimal solution. For instance, if penalties are $\hat{H} = 10$ y $H_i = \tilde{H}_i = 1, \ i = 2, \ldots, 12$, then it took 6 minutes 27 seconds to solve the VRPSTW. Similarly, if penalties are $\hat{H} = 1$ y $H_i = \tilde{H}_i = 1, \ i = 2, \ldots, 12$, then 23 minutes 12 seconds are required to get an optimal solution.

We have also solved problems with up to 15 customers, but the computational time needed to get an optimal solution increases very quickly. For instance, considering always that all penalties are equal to 1, when the network involves 13 nodes it took 23 minutes to get an optimal solution; if there are 14 nodes more than 1 hour and 30 minutes is needed; if there are 15 nodes more than 9 hours are needed; and, finally, it took more than 53 hours to solve an example with 16 nodes. Clearly, this is not very appealing when using the model to handle real systems.

When the fleet of vehicles is not homogeneous, the number of 0-1 variables involved is so large that getting an exact optimal solution of the model is not efficient from the point of view of practical applications. For example, the resolution of a classical VRP to compute Z in a model with three vehicles, which involves 434 variables from which 399 are binary variables, was abort after more than 1 hour and 30 minutes of running time.

Therefore, future work should consider an approach that combines heuristics and optimization, which allows us to obtain optimal solutions or approximate optimal solutions in an acceptable amount of computational time. This will make possible to design a decision support system which can be used in real time to determine delivery routes in more complex real life applications. We also think that a two-phase approach which, (1) computes feasible routes from the point of view of driving time involved and, (2) takes into account preferences on time windows, might be efficient. Notice that in most of real systems with long distances between customers, routes do not contain a lot of customers, which allows us to infer that the set of feasible routes can be efficiently obtained.
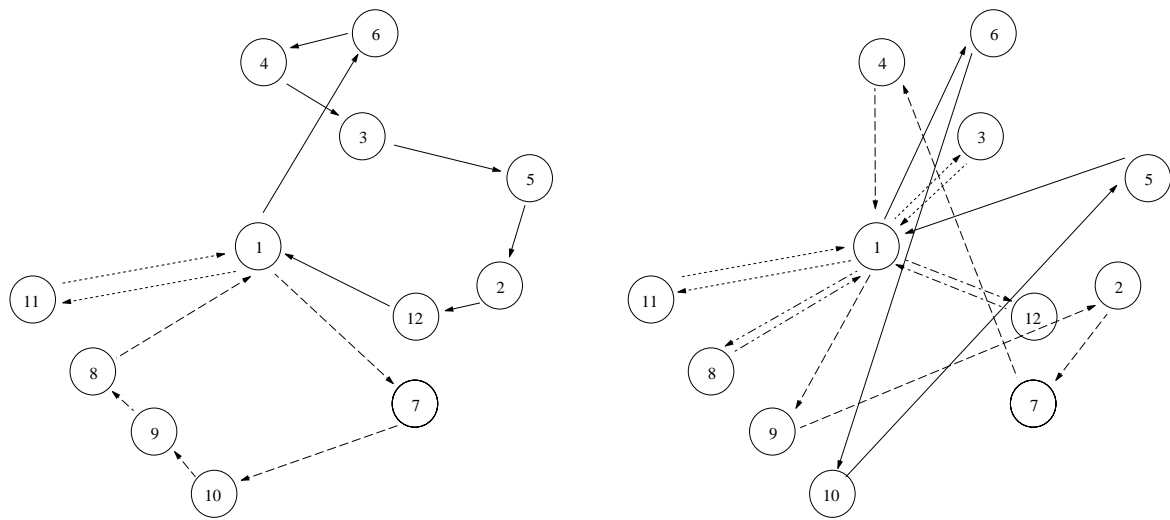
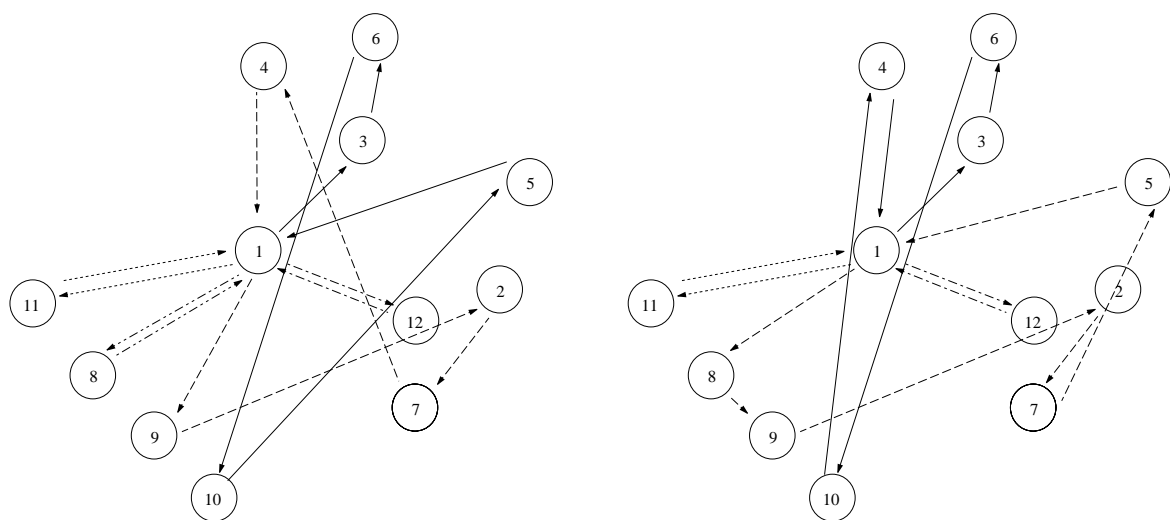Figure 1: (a) Minimum cost routes.   (b) Routes with hard time windows.

Figure 2: (a) Routes with soft time windows.   (b) Routes with at most 4 vehicles.

Acknowledgements

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