Monografías del Semin. Matem. García de Galdeano. 27: 121–125, (2003).

# Regulatory distorsions in a competitive banking industry: stockholder commercial versus mutual banks

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#### Abstract

This paper aims to present a theoretical analysis of strategic competition in retail banking when some of the firms of the industry show profit maximising behaviour (commercial banks) and some other show expense preference conduct (mutual banks). Specifically, we study the impact of imposing a prudential regulation (minimum capital ratio) on those different types of banks. This works build upon recent strategic banking studies ([1],[2] and [3]) which consider, from a mathematical perspective, the competitive behaviour of banking firms in highly liberalized markets but with a regulatory distortion. In order to get this aim, it will be necessary to solve some problems posed in the field of optimisation, particularly on maximisation of functions subjected to some restrictions.

**Keywords**: Optimisation, utility/objective function, duopoly. **AMS Classification**: 62P20, 91B02, 91B50, 91B74.

# 1 Hypotheses and notation

Commercial banks and mutual banks lend funds to firms and regulators impose capital requirements. We consider a minimum capital ratio regulation, that is, we assume that, for each 1 monetary unit (m.u.) raised,  $\delta \in (0, 1)$  are funded by deposits and  $1 - \delta \in (0, 1)$ are funded by equity capital. On extending a loan, a financial institution (commercial bank or mutual bank) must choose its per-loan monitoring level,  $m \in (0, 1)$ . It will choose the monitoring level to minimise the total cost of the loan, which includes both potential credit losses from defaults and direct monitoring costs. Expected credit losses are a function of the monitoring level, L(m), which is decreasing and concave in m. We assume that credit losses are defined as L(m) = 1 - m. The direct costs of monitoring are given by the increasing and convex function  $V(m) = \alpha m^2$ , where  $\alpha$  is a positive constant. Thus, we can express the expected cost of issuing a one m.u. loan as

$$C(m) = (1 - \delta)(1 - m) + \alpha m^2.$$
 (1)

The first step is to choose m to minimise (1). We obtain:

$$m^* = \frac{1-\delta}{2\alpha}$$
 and  $C(m^*) = (1-\delta) - \frac{(1-\delta)^2}{4\alpha}$ .

The model we present considers a bank, represented by subindex*b*, and a savings institution, represented by subindex *s*. Bothinstitutions compete for loans in a Cornout duopoly and chooses aquantity of loans to produce, given by  $Q_s$  (for the saving bank) and  $Q_b$  (for the commercial bank). The optimal monitoring levels for the commercial bank and for the savings bank are denoted by  $m_b^*$  and  $m_s^*$ , respectively. At the optimal level, therefore, the per-unit expected costs of extending a m.u. loan, are denoted  $C_b$  and  $C_s$ . Following [2], we will consider the following notation and hypotheses:

- \* The aggregate demand for loans over the period is given by  $\Omega$ .
- \* We suppose  $Q_b = K_b L_b$  and  $Q_s = K_s L_s$ , where  $K_s (K_b)$  is the average and marginal productivity of the savings bank (commercial bank) and  $L_s (L_b)$  is the number of workers of the savings bank (commercial bank). We assume that both firms are equally efficient, that is,  $K_b = K_s := K$ .
- $\star$  The cost of labour per worker is  $\omega$  in both institutions.
- \* We denote  $c = \frac{\omega}{K}$ .
- $\star$  The per-unit price of a 1 m.u. loan, P, is determined by the inverse demand function:

$$P(\Omega, Q) = \Omega - Q,$$

where  $Q = Q_b + Q_s$  is the total quantity produced.

★ Demand for loans is such that there is room for both types of banks to compete. That is, the demand structure allows for positive profits to both competitors.

### 2 The model

With the previous notation, the economic profit of each institution is given by:

$$\Pi_b = (\Omega - Q_b - Q_s)Q_b - C_bQ_b - \frac{wQ_b}{K}$$
$$\Pi_s = (\Omega - Q_b - Q_s)Q_s - C_sQ_s - \frac{wQ_s}{K}$$

Banks are assumed to maximise profits, but the objective function of savings institutions is unclear. The literature on expense preference behaviour assumes that the loose assignment of property rights in institutions such as savings banks allows the managers of such institutions to choose their own preference function in place of profit maximisation, subject to constraint of not having operating losses. For a savings bank, it is assumed that manager's utility function will depend on profits and labour expenses,  $U_s(\Pi_s, E_s)$ , where  $E_s = \omega L_s$  and  $\frac{\partial U_s}{\partial \Pi_s} > 0$  and  $\frac{\partial U_s}{\partial E_s} > 0$ . We assume that  $U_s$  is linear in  $\Pi_s$  and  $E_s$ , i.e.:

$$U_s = \Pi_s + \theta \omega L_s,$$

where  $\theta$  is a positive parameter (because utility increases with labour expenses). Notice that if  $\theta = 0$ ,  $U_s = \Pi_s$  and if  $\theta = 1$ ,  $U_s = \omega \mathbf{L}_s$ . Therefore, the utility function  $U_s$  implies that the savings banks is an institution in between a capitalist firm ( $\theta = 0$ ) and a worker's cooperative ( $\theta = 1$ ) and hence, we suppose  $\theta \in (0, 1)$ .

With these considerations we look for the Nash equilibrium solution to the following optimisation problem:

$$\max_{Q_s} U_s = (\Omega - Q_b - Q_s)Q_s - C_sQ_s - \frac{wQ_s}{K} + \frac{\theta\omega Q_s}{K}$$
$$\max_{Q_b} \Pi_b = (\Omega - Q_b - Q_s)Q_b - C_bQ_b - \frac{\omega Q_b}{K}$$

### 3 Main results

FIRST CASE: Costs are equal for both institutions  $(C_b = C_s (:= C))$ 

- Observe that  $C_b = C_s$  is equivalent to  $m_b^* = m_s^*$ .
- The equilibrium solution is:

$$Q_b^* = \frac{1}{3}(-c - C + \Omega - c\theta); \quad Q_s^* = \frac{1}{3}(-c - C + \Omega + 2c\theta)$$
$$\Pi_b^* = \Pi_b(Q_b^*, Q_s^*) = \frac{1}{9}(c + C - \Omega + c\theta)^2$$
$$\Pi_s^* = \Pi_s(Q_b^*, Q_s^*) = -\frac{1}{9}(c + C - \Omega + c\theta)(-C + \Omega + c(-1 + 2\theta))$$

)

• It is easy to show that  $Q_b^* < Q_s^*$  and  $\Pi_b^* < \Pi_s^*$ . In other words, for the expense preference solution, savings banks capture a larger share of the market and earn higher profits than commercial banks. So the theory is consistent with the empirical evidence that savings banks gain share and obtain more profits than commercial banks.

$$\left|\frac{\partial \Pi_b^*(C)}{\partial C}\right| < \left|\frac{\partial \Pi_s^*(C)}{\partial C}\right|,$$

so savings banks profits are more sensitive than commercial banks profits to changes in costs. The implication of this result is that, even when regulators can ensure that their actions affect all banks identically in terms of costs, the savings banks will suffer a greater absolute loss in profits.

#### SECOND CASE: Commercial bank costs>Savings bank costs $(C_b > C_s)$

• The equilibrium solution is:

$$Q_b^* = \frac{1}{3}(-c - 2C_b + C_s + \Omega - c\theta); \quad Q_s^* = \frac{1}{3}(-c + C_b - 2C_s + \Omega + 2c\theta)$$
$$\Pi_b^* = \Pi_b(Q_b^*, Q_s^*) = \frac{1}{9}(c + 2C_b - C_c - \Omega + c\theta)^2$$
$$\Pi_s^* = \Pi_s(Q_b^*, Q_s^*) = -\frac{1}{9}(c - C_b + 2C_s - \Omega + c\theta)(C_b - 2C_s + \Omega + c(-1 + 2\theta))$$

• In this case it can be also shown that  $Q_b^* < Q_s^*$  and  $\Pi_b^* < \Pi_s^*$ . That is, the institution that has the cost advantage (in this case, the savings institution), obtains a larger share of the market and enjoys a higher expected profit.

$$\left|\frac{\partial \Pi_b^*(C_s^*, C_b^*)}{\partial C_b^*}\right| < \left|\frac{\partial \Pi_s^*(C_s^*, C_b^*)}{\partial C_s^*}\right|$$

This shows that although the savings bank has the cost advantage, it is more sensitive to changes in costs.

# THIRD CASE: Commercial bank costs < Savings bank costs ( $C_b < C_s$ )

• The equilibrium solution is:

$$Q_b^* = \frac{1}{3}(-c - 2C_b + C_s + \Omega - c\theta); Q_s^* = \frac{1}{3}(-c + C_b - 2C_s + \Omega + 2c\theta)$$
$$\Pi_b^* = \Pi_b(Q_b^*, Q_s^*) = \frac{1}{9}(c + 2C_b - C_s - \Omega + c\theta)^2$$
$$\Pi_s^* = \Pi_s(Q_b^*, Q_s^*) = -\frac{1}{9}(c - C_b + 2C_s - \Omega + c\theta)(C_b - 2C_s + \Omega + c(-1 + 2\theta))$$

Now we distinguish the following cases:

- If  $C_s \ge C_b + c$ , we obtain  $Q_b^* > Q_s^*$  and  $\Pi_b^* > \Pi_s^*$ .
- If  $C_s < C_b + c$ , we obtain:
  - a) If  $\theta \leq \frac{C_s C_b}{c}$ , then  $Q_b^* > Q_s^*$  and  $\Pi_b^* > \Pi_s^*$ .

b) If  $\theta > \frac{C_s - C_b}{c}$ , then  $Q_b^* < Q_s^*$  but there is no relationship between profits, as the following examples show: let  $C_b = 99, C_s = 101, c = 10, \Omega = 125$  and  $\theta = 0.5$ . Then,  $C_s < C_b + c, \theta > \frac{C_s - C_b}{c}, Q_b^* = 13/3, Q_s^* = 22/3$  and the per-unit price  $P(\Omega, Q) = \Omega - Q$  is positive. In this case,  $\Pi_b^* = 169/9$  and  $\Pi_s^* = 154/9$ . However, for  $\theta = 0.8$ , we have  $\theta > \frac{C_s - C_b}{c}, Q_b^* = 10/3, Q_s^* = 28/3$  and thus, the per-unit price is also positive. But now  $\Pi_b^* = 100/9$  and  $\Pi_s^* = 112/9$ .

# 4 Conclusions

preference behaviour does Expense not appear to prevent mutual (savings) banks from gaining market share and obtaining higher profits in certain European countries (Spain, Germany). Using a model where profit maximisers (commercial banks) and utility maximisers (mutual banks) compete in an oligopolistic (loan) market with regulatory distortions (a capital adequacy requirement), we show that savings banks obtain higher market share and higher profits than commercial banks in, at least, two out of the three cases analysed. These two cases are: (1) when both types of institutions enjoy the same per-unit expected cost of issuing a loan; (2) and when mutual banks have a lower per-unit cost of issuing a loan. Only when commercial banks have lower costs, they are shown to have greater market share and profits, but not in all cases since, even when profit-maximising banks show a cost advantage, under certain circumstances  $[\theta > \frac{C_s - C_b}{c}]$ , mutual banks may show greater competitiveness in terms of market share.

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