Universidad Zaragoza
1542

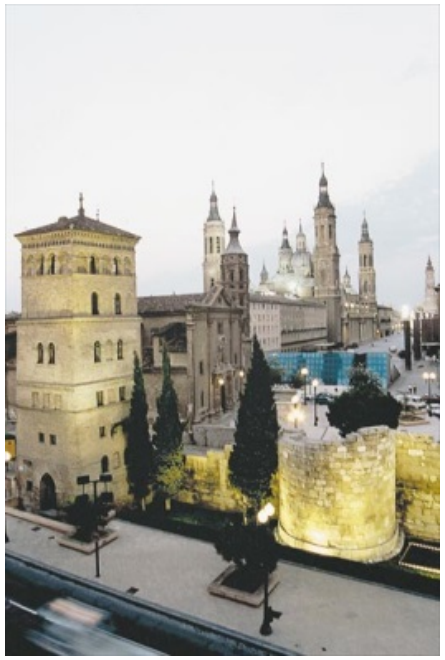
Sixteenth International Conference

Zaragoza-Pau

on Mathematics and its Applications

Jaca, 7-9 September 2022

UNIVERSITÉ DE PAU ET DES PAYS DE L'ADOUR



We are very pleased to welcome you to the *Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications* and we are very grateful for your participation. Inside you will find the programme, the list of participants and the summaries of the different communications.

Owing to the number of talks, we have distributed most of them in three parallel sessions. We note that this year there are nine plenary conferences and eleven mini-symposia, five of them co-organized by colleagues from the Université de Pau and the Universidad de Zaragoza.

This event would not have been possible without the financial and material support of the Université de Pau and the Universidad de Zaragoza. We are grateful to both these Institutions, but without forgetting the invaluable support given by the Pyrenean Work Community which has stimulated across the four regions of Aquitaine, Aragón, Midi-Pyrénées and Navarra the close collaboration between the *Laboratoire de Mathématiques et de leurs Applications de l'Université de Pau*, the *Departamentos de Matemática Aplicada, de Matemáticas* and *de Métodos Estadísticos de la Universidad de Zaragoza*, the *Centre de Recherche CEREMATH de l'Université de Toulouse I* and the *Departamento de Estadística, Informática y Matemáticas de la Universidad Pública de Navarra*.

In like manner, we would like to make special mention of the support received from the Government of Aragón and the European Social Fund and from the IUMA de la Universidad de Zaragoza (*Instituto Universitario de Matemáticas y Aplicaciones*). We would like to thank also the *Institut Carnot ISIFoR*, the *Fédération IPRA* of Pau (*Institut Pluridisciplinaire de Recherche Appliquée*) and the *CNRS* (*Centre National de la Recherche Scientifique*) for their support.

Finally, our sincere gratitude to Pedro Mateo and Luis Rández, from the Universidad de Zaragoza, and to Juan José Torrens, from the Universidad Pública de Navarra, for their invaluable help in organizing the web and editing the abstracts.

Jaca, 7th September 2022
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- Enrique Artal, IUMA, Department of Mathematics, UZ
- Gilles Carbou, Lab. Mathématiques et leurs Applications, UPPA
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Programme

Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications

Wednesday September 7th

8:30-9:30 REGISTRATION

PLENARY TALKS Sala conferencias
9:30-10:15 S. Molins
10:15-11:00 F.J. Marcellán

11:00-11:45: Coffee Break / Poster session

MS05 Applications of fractional differential systems Sala conferencias
11:45-12:10 — A. Szafranska
12:10-12:35 — J. Cresson
12:35-13:00 — P.J. Miana
13:00-13:25 — J. Oliva

MS01 Parabolic problems in natural sciences, engineering and economics Aula 3
11:45-12:15 — P. Takáč
12:15-12:45 — B. Alziary
12:45-13:15 — M. Cuesta

Session 1 Aula 1
11:45-12:10 — A. Mayora
12:10-12:35 — C. Mayora
12:35-13:00 — Y. Tahraoui

13:45-14:45: Lunch

MS03 PDEs and fluid mechanics Sala conferencias
15:20-15:50 — M.A. Rodriguez-Bellido
15:50-16:20 — R. Lewandowski
16:20-16:50 — T. Chacón

MS04 Dynamics in epidemics, populations and cell models Aula 3
15:20-15:50 — F. Avram
15:50-16:20 — R. Barrio
16:20-16:50 — J. Jover
16:50-17:20 — A. Lozano

Session 2 Aula 1
15:20-15:45 — C. Ferreira
15:45-16:10 — E. Pérez
16:10-16:35 — F. Nudo
16:35-17:00 — H. Orera
17:00-17:25 — C.E. Bréhier

17:25-18:00: Coffee Break / Poster session

PLENARY TALK Sala conferencias
18:00-18:45 C. de Coster

19:15-: Official opening ceremony

20:00-: Reception: Residencia universitaria de Jaca

Thursday September 8th

MS02 (Block I) Mathematical modelling and numerical analysis for CFD Sala conferencias
9:00- 9:30 — J. Nuñez de la Rosa
9:30-10:00 — I. Echeverribar
10:00-10:30 — S. Martínez
10:30-11:00 — P. Solan

MS10 Applications of mathematical programming Aula 3
9:00- 9:30 — M. Cildoz
9:30-10:00 — P. Mateo
10:00-10:30 — J.A. Iranzo

Session 3 Aula 1
9:00- 9:25 — E. Lanchares
9:25- 9:50 — V. Lanchares
9:50-10:15 — L. Floría
10:15-10:40 — K. Hariz-Belgacem
10:40-11:05 — R. Safi

11:00-11:45: Coffee Break / Poster session

PLENARY TALKS Sala conferencias
11:45-12:30 J. Pacheco
12:30-13:15 D. Lannes

MS11 (Block I) Algebra & Geometry Aula 1
11:45-12:25 — S. Marchesi
12:30-13:10 — J. Vallès

13:45-14:45: Lunch

MS07 Reactive flows from the pore-scale to the reservoir-scale Sala conferencias
15:15-15:45 — S. Molins
15:45-16:15 — S. Perez
16:15-16:45 — E. Ahusborde

MS08 Probability & Statistics I Aula 3
15:15-15:45 — J.A. Adell
15:45-16:15 — A. Camón
16:15-16:45 — C. Paroissin
16:45-17:15 — M. Pereda

Session 4 Aula 1
15:15-15:40 — J. Ghantous
15:40-16:05 — A. Pé
16:05-16:30 — J. Zaratiegui
16:30-16:55 — C. Caballero
16:55-17:20 — I. Gómez

17:15-18:00: Coffee Break / Poster session

PLENARY TALKS Sala conferencias
18:00-18:45 Y. Privat

MS09 Probability & Statistics II Aula 3
17:30-18:00 — G. Badía
18:00-18:30 — M. Gómez
18:30-19:00 — S. Mercier
19:00-19:30 — C. Sangüesa

MS11 (Block II) Algebra & Geometry Aula 1
18:00-18:35 — C. Alquézar
18:40-19:15 — M. Manouras
19:20-20:00 — E. Elduque

21:00-: Conference dinner: Restaurant Cobarcho

Friday September 9th

MS02 (Block II) Mathematical modelling and numerical analysis for CFD Sala conferencias
9:00- 9:30 — R. Zelada
9:30-10:00 — P. Vallés
10:00-10:30 — J. Mairal
10:30-11:00 — A. Navas

MS06 Discretizations and convergence of methods for IBVP Aula 3
9:00- 9:30 — B. Cano
9:30-10:00 — G. Hauke
10:00-10:30 — J.C. Jorge
10:30-11:00 — I. Royo

Session 5 Aula 1
9:00- 9:25 — S. Boukassa
9:25- 9:50 — I. Boussetouan
09:50-10:15 — M. Kassan

11:00am-11:45am: Coffee Break

PLENARY TALKS Sala conferencias
11:45-12:30 E. Ferrer
12:30-13:15 M. Cuesta

MS11 (Block III) Algebra & Geometry Aula 1
11:45-12:25 — M. Herradón
12:30-13:10 — E. León

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Random linear operators arising from piecewise linear interpolation on the unit interval

José A. Adell[1], Daniel Cárdenas-Morales[2]

SUMMARY

We introduce a sequence of random linear operators arising from piecewise linear interpolation at a set of random nodes on the unit interval. We show that such operators uniformly converge in probability to the target function, providing at the same time rates of convergence in terms of the Ditzian-Totik modulus of smoothness. Analogous results are shown for their deterministic counterparts, derived by taking expectations of the aforementioned random operators. Special attention is paid to the case in which the random nodes are the uniform order statistics. This allows us to compare the speed of convergence in the case at hand with that concerning the random and the deterministic Bernstein polynomials.

Keywords: random linear operator, random Bernstein polynomials, uniform convergence in probability, Ditzian-Totik modulus of smoothness, uniform order statistics.

AMS Classification: 41A25, 60E05

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Numerical simulation of Thermo-Hydro-Chemical processes for subsurface problems

Etienne AHUSBORDE[1] Fabrizio CROCCOLO[2], Nicolas PILLARDOU[1,2]

SUMMARY

Reactive multiphase multicomponent flows in porous media are involved in many applications related to subsurface environment and energy issues. We can mention non exhaustively production of geothermal enrergy, geological sequestration of gas (H_2, CO_2, CH_4) or nuclear waste management.
Such flows are governed by Thermo(T), Hydro (H), Chemical (C) phenomena. More precisely they are modelled by a set of highly nonlinear system of degenerate partial differential equations (describing a multiphase compositional flow through mass and energy conservation laws) coupled with algebraic and/or ordinary differential equations (related to geochemical model) requiring special numerical treatment. The numerical strategies for solving this system are divided into two main categories: the global implicit and the sequential approaches. The global implicit approach solves one global nonlinear system gathering all equations at each time step while for the sequential approach, flow and reactive transport are solved sequentially at each time step.
In the framework of the parallel platform DuMu^X [1], we have developed an implemented a sequential [2] and a global implicit scheme [3] to deal with isothermal reactive multiphase flows. In this work, both strategies have been extended to non-isothermal flow and validated by several test cases including High Performance Computing. A comparison between both strategies for a three dimensional scenario of geological storage of CO_2 will be presented.

Keywords: Multiphase flow, reactive, porous medium, finite volume, HPC.

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An incremental Non-Dominated Sorting framework based on Irreducible Domination Graphs

I. Alberto[1], D. Lahoz[1], P.M. Mateo[2]

SUMMARY

Non-Dominated Sorting process, NDS, plays an important role in Pareto based Evolutionary Multi-Objective Optimization Algorithms and it is one of the most time consuming tasks, mainly when steady-state Evolutionary Algorithms are considered. In this work we present a general framework to carry out the NDS process and three implementations based on a modification of the Irreducible Domination Graph structure, IDG, presented in Alberto and Mateo (2004) [1] for accomplishing this task.

Our algorithms are benchmarked against other NDS algorithms focused on incremental updating of Pareto layers ([2], [3], [4]). The experiments accomplished show that the implementation of the proposed algorithms reduce, in general, the number of Pareto comparisons as well as the time needed when compared with the competitors.

Keywords: Non-dominated Sorting, Multi-objective Optimization, Graphs

AMS Classification: 90-08,68W50, 90C29

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Efficient time integration of nonlinear partial differential equations by means of Rosenbrock methods

I. Alonso-Mallo[1], **B. Cano**[2]

SUMMARY

We avoid as as much as possible the order reduction of Rosenbrock methods when they are applied to nonlinear partial differential equations by means of a similar technique to the one used previously by us for the linear case [1]. For this we use a suitable choice of boundary values for the internal stages. The main difference from the linear case comes from the difficulty to calculate those boundary values exactly in terms of data. In any case, the implementation is cheap and simple since, at each stage, just some additional terms concerning those boundary values and not the whole grid must be added to what would be the standard method of lines.

Keywords: Nonlinear partial differential equations, Rosenbrock method, order reduction

AMS Classification: 65M12, 65M20

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Effective computation of the Sullivan model of a topological space and its applications

Carlos Alquézar Baeta

SUMMARY

Due to Sullivan, given a topological space X, it is known theoretically how to construct a commutative differential graded algebra, called the Sullivan model of the space, that is quasi-isomorphic to the normalized singular cochain algebra, $C^*(X)$. This object is an algebraic invariant of the space, and in some cases, it contains topological information of X. Here, we present an effective algorithm (with a concrete implementation) for the computation of the Sullivan model of a given topological space.

In this talk, we will: (1) introduce the notion of the Sullivan model of a space, (2) explain in which cases this model captures the topological information we are interested in, and (3) we will present an effective algorithm (with a concrete implementation) for the computation of the Sullivan model of this kind of topological spaces.

Keywords: Sullivan models, Rational Homotopy, Simplicial Complexes, Computation

AMS Classification: 55P62, 55U10, 55-08

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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The Heston stochastic volatility model has a boundary trace at zero volatility

Bénédicte Alziary[1], Peter Takáč[2]

SUMMARY

We establish boundary regularity results in Hölder spaces for the degenerate parabolic problem obtained from the Heston stochastic volatility model in Mathematical Finance set up in the spatial domain (upper half-plane) $\mathbb{H} = \mathbb{R} \times (0, \infty) \subset \mathbb{R}^2$. Starting with nonsmooth initial data $u_0 \in H$, we take advantage of smoothing properties of the parabolic semigroup $\mathrm{e}^{-t\mathcal{A}}\colon H \to H$, $t \in \mathbb{R}_+$, generated by the Heston model, to derive the smoothness of the solution $u(t) = \mathrm{e}^{-t\mathcal{A}}u_0$ for all $t > 0$. The existence and uniqueness of a weak solution is obtained in a weighted Hilbert space $H = L^2(\mathbb{H}; \mathfrak{w})$.

Keywords: first word, second word, third word,...

AMS Classification: 35B65, 35K65, 35K15, 91G80

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Can the Lotka-Volterra and mass-action canonical forms for kinetic systems be used to study SIR-PH type epidemic models?

Florin Avram

SUMMARY

We propose exploiting the Lotka-Volterra and mass-action canonical forms for studying epidemic models, and illustrate the results obtained for a SIR-PH type example.

Keywords: Essentially non-negative polynomial ODE systems, Lotka-Volterra canonical form, chemical reaction networks, mass-action kinetics, steady states, multistationarity

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On the residual lifetime and inactivity time in mixtures

F.G. Badía[1], **M.D. Berrade**[2]

SUMMARY

In this paper we study the ageing characteristics in mixtures of distributions, providing characterizations for its derivatives that explain the smooth behaviour of the mixture. The classical preservation results for the reversed hazard rate, mean residual life and mean inactivity time are derived under a different approach than that in previous works. We focus on the variance of both the residual life and inactivity time in mixtures, obtaining some preservation properties. We also state conditions for weak and strong bending properties for the variance of the residual life and the inactivity time in mixtures.

Keywords: mixture, residual life variance, inactivity time variance, ageing class, bending property

AMS Classification: 62N05, 60E15, 60K10

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Condition-based maintenance and age replacement in a system with two stochastically dependent components

F.G. Badía[1], **M.D. Berrade**[1], **M. Gómez**[1]

SUMMARY

The failure of some components in complex systems may affect the state of the rest this stochastic dependence should be considered when designing a maintenance model. In this paper we present a model for inspection and maintenance of a system with two components stochastically dependent. Component 1 may be in one of two states, good and failed. Component 2 can present one of three states: good, defective and failed. Thus, the time to failure of component 2 is modeled in two stages, from good to defective and from defective to failure (delay time, Christer [1]). Failures of component 1 and defective states of component 2 are unrevealed, that is, they are detected by inspection. Failures of component 2 can be considered as hard failures in the sense that the full system with the two components has to be replaced and a high cost is derived. In addition they are revealed at the very moment they take place. Aiming at detecting failures of component 1, it is inspected every T units of time. We propose a condition-based inspection for component 2 since component 2 is only inspected in case that component 1 is found to be failed. If it is in the defective state, the full system is replaced by a new one. If not, then only component 1 is replaced and the systems keeps on functioning until the following inspection, hard failure or preventive maintenance after M inspections, at MT, whichever comes first. We assume that failures of component 1 may affect the reliability of component 2. Thus, the baseline hazard rate $\lambda_0(t)$, corresponding to the delay-time in component 2, changes to $\lambda_1(t)$, with $\lambda_0(t) < \lambda_1(t)$, in case that component 1 fails while component 2 is defective. Hence, the stochastic dependence implies that component 1 degrades faster as a result of failures in component 1. The model leads to a cost function with two decision variables: the inspection interval, T, and the maximum number of inspections, M, before renewal. In order to check the advantages of this model for practical purposes, the optimum policy, $T^\star$ and $M^\star$, minimizing the cost function is compared with that derived from a model without age replacement ($M = \infty$).

Keywords: age replacement, condition-based maintenance, stochastic dependence

AMS Classification: 90B25, 60K10, 62N05

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Estimating the distance between the invariant manifolds of L_3 in the RCP3BP usign high precision methods

Inmaculada Baldomá[1], Ernesto Lanchares, Mercè Ollé[2]

SUMMARY

A short time ago, in [1] and [2] an asymptotic for the distance of the invariant manifolds of L_3 in the RCP3BP when the mass parameter tends to zero was proven. In this work, we will approximate the values of the constants involved in the formula. To that end, we numerically estimate the distance between the manifolds using high precision methods and substitute the obtained values in the aforementioned formula. Moreover, we will use the inner equation of the problem to give better approximations of some of the constants involved. In the process we develop new algorithms and adapt preexisting methods to work with high precision floating point numbers.

Keywords: invariant manifold, Parameterization method, inner equation

AMS Classification: 34E15, 37D10, 37M21

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A dynamical mechanism for generation of arrhythmogenic early afterdepolarizations in cardiac myocytes

Roberto Barrio[1], Jorge Alberto Jover-Galtier[1], M. Ángeles Martínez[1], Ana Mayora-Cebollero[1], Sergio Serrano[1], Rubén Vigara[1]

SUMMARY

Early Afterdepolarizations (EADs), which are voltage oscillations in cardiac action potential during the repolarization phase, are linked to the appearance of cardiac arrhythmias and other heart conditions. In this presentation we analyze the dynamical mechanisms underlying the formation of arrhythmogenic early afterdepolarizations (EADs) in two mathematical models of cardiac cellular electrophysiology: a biophysically detailed model of a ventricular myocyte with a large number of state variables (which allow a more faithful reproduction of experimental observations) and a low dimensional model, more suitable for theoretical analysis. Based on a comparison of the two models, with detailed bifurcation analysis using continuation techniques in the simple model and numerical explorations in the complex model, we propose a conjectured scheme involving a hysteresis mechanism with the creation of alternans and EADs in the unstable branch. This theoretical scheme fits well with electrophysiological experimental data on EAD generation and hysteresis phenomena [1, 2, 3].

Keywords: Cardiac dynamics, bifurcations

AMS Classification: 34C60, 92B05

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Sufficient conditions for some stochastic orders of discrete random variables with applications in reliability

Félix Belzunce[1], **Carolina Martínez-Riquelme**[1], **Magdalena Pereda**[2]

SUMMARY

In this paper we focus on providing sufficient conditions for some well-known stochastic orders in reliability but dealing with the discrete versions of them, filling a gap in the literature since there is just one paper on this topic. In particular, we find conditions based on the unimodality of the ratio of the mass probability functions for the comparison on some stochastic orders of two discrete random variables. These results have interest in comparing discrete random variables because the sufficient conditions are easy to check when there are no closed expressions for the survival functions, which occurs in many cases. In addition, the results are applied to compare several parametric families of discrete distributions.

Keywords: Stochastic orders, Discrete distributions, Unimodality, Panjer, generalized Poisson.

AMS Classification: 60E05, 60E15

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Weak solution for a ferrofluid flow model

Saliha Boukassa[1], Chérif Amrouche[2]

SUMMARY

We prove the existence of solution for a model of differential system introduced by Shliomis. It consists of the Navier-Stokes equations, the magnetization equation and the magnetostatic equations, see [3]. The equations describe the stationary flow of an incompressible ferrofluid submitted to the action of an external magnetic field. The external magnetic field induces a demagnetizing field and a magnetic induction. The magnetization equation is of Bloch-Torrey type. We proceed by linearisation and application of Leray-Schauder fixed point Theorem. We give regularity results in L^p-Theory.

Keywords: Ferrofluid, Shliomis, L^p-Theory.

AMS Classification: 35E05, 35Qxx

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On Stokes and Navier-Stokes equations with Navier and Dirichlet boundary conditions

Imane Boussetouan[1], Chérif Amrouche[2]

SUMMARY

In this work, we consider two types of mixed boundary conditions associated to the Stokes and Navier-Stokes systems. Firstly, we study the Stokes equation with Dirichlet boundary condition on some part of the boundary and Navier-type boundary condition on the remaining part [3]. We prove the existence and uniqueness of weak and strong solutions of the corresponding problem in the Hilbert setting [1]. Then, we generalize our solutions to the L^p-theory, by means of a bootstrap argument. Secondly, we assume that the non-homogeneous Navier boundary condition is prescribed on one part of the boundary and Dirichlet boundary condition on the other part. We prove the existence and uniqueness of the solution in $\mathbf{W}^{1,p}(\Omega)$ and $\mathbf{W}^{2,p}(\Omega)$ by taking into account the regularity of the friction coefficient, considered as a function [2]. Finally, we extend the obtained results to the stationary Navier-Stokes system by using some classical arguments.

Keywords: Navier-Stokes equation, Navier boundary condition, mixed boundary conditions, L^p theory.

AMS Classification: 35J05, 76D03

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Analysis of a modified Euler scheme for SPDEs

Charles-Edouard Bréhier

SUMMARY

A novel integrator to approximate solutions of parabolic semilinear stochastic evolution equations driven by space-time white noise

$$dX(t) = -\Lambda X(t)dt + F(X(t))dt + dW(t)$$

is presented. The proposed modified Euler scheme, introduced in [1], is written as

$$X^\tau_{n+1} = \mathcal{A}_\tau\big(X^\tau_n + \tau F(X^\tau_n)\big) + \mathcal{A}_\tau\sqrt{\frac{\tau}{2}}\Gamma_{n,1} + \mathcal{B}_\tau\sqrt{\frac{\tau}{2}}\Gamma_{n,2},$$

where $\tau = T/N$, $\mathcal{A}_\tau = (I+\tau\Lambda)^{-1}$ $\Gamma_{n,1}, \Gamma_{n,2}$ denote independent cylindrical Gaussian random variables, and the linear operator $\mathcal{B}_\tau$ satisfies the condition $\mathcal{B}_\tau\mathcal{B}^\star_\tau = (I+\tau\Lambda)^{-1}$.

The objective of the talk is to present the main improvements when the modified Euler scheme is used instead of the standard method (given by $\mathcal{B}_\tau = \mathcal{A}_\tau$).

- The spatial regularity is preserved at all times, for any value of the time-step size τ.
- When $F = 0$, the Gaussian invariant distribution ν of the Ornstein–Uhlenbeck process $\big(X(t)\big)_{t\geq 0}$ is preserved by the numerical scheme, for any value of the time-step size τ.
- If $F = -DV$ and if an appropriate ergodicity condition is satisfied, one can approximate the invariant Gibbs distribution given by

  $$d\mu_\star(x) = Z^{-1}\exp(-2V(x))d\nu(x)$$

  of the process in the total variation distance: for any τ, the scheme admits a unique invariant distribution μ^τ_∞, and for all $\kappa \in (0, \frac{1}{2})$, there exists $C_\kappa \in (0,\infty)$ such that

  $$d_{\mathrm{TV}}(\mu^\tau_\infty, \mu_\star) \leq C_\kappa \tau^{\frac{1}{2}-\kappa}.$$

Keywords: stochastic partial differential equations, numerical approximation, invariant distribution

AMS Classification: 65C30,60H35,60H15

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Implicit and implicit-explicit Lagrange-projection exactly well-balanced finite-volume schemes for the one-dimensional shallow-water system

C. Caballero-Cárdenas[1], M. J. Castro[1], T. Morales de Luna[1], M. L. Muñoz-Ruiz[1]

SUMMARY

In this work we present implicit and implicit-explicit first and second order numerical approximations of the shallow-water equations based on a Lagrange-Projection type finite volume strategy. This scheme can be interpreted as a two-step algorithm consisting in first solving the shallow water system in Lagrangian coordinates, which is known as the Lagrangian step, and then projecting the results in Eulerian coordinates, which is known as the Projection step. For the Lagrangian step we propose two different implicit versions: one fully implicit and one implicit-explicit, depending on how how the source term is treated. The Projection step will always be done explicitly.

By following this strategy, the acoustic and the transport phenomena can be decoupled and this allows us to design large time step schemes in which the CFL restriction is based on the slower transport waves and not on the acoustic ones. In this work we follow the strategy described in [1, 3] to define the Lagrange-Projection scheme and [2] to ensure its well-balanced character.

Keywords: Lagrange-Projection strategy, IMEX schemes, well-balanced, shallow water equations

AMS Classification: 35Lxx, 65Mxx

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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An effective algorithm for balancing the assignment of customers among clusters

Herminia I. Calvete[1], Carmen Galé[2], José A. Iranzo[3]

SUMMARY

A common strategy of logistics companies is to group their customers into clusters, so that they can better manage their resources and workload. The definition of clusters is, in general, a tactical decision, maintained over time in the medium term, based on geographic boundaries, levels of regular demand, etc. However, on a day-to-day basis, the number of customers requiring service in each cluster varies. As a result, the daily number of customers needing to be visited in each cluster fluctuates, which can lead to imbalances between clusters.

In this work, we study a problem posed by a consulting firm that provides services to the logistics industry. The objective is to study how some customers can be removed from their pre-assigned cluster and reassigned to a different cluster, so that the final number of customers in the clusters is balanced. Three different approaches are proposed to address such balancing. Customer transfers should be done, preferably, in such a way that they affect customers that are originally close to the area to whose cluster they are to be reassigned. In addition, this problem should be solved in a short period of time, so that it can be applied when planning the daily workload.

This problem has been modeled using lexicographic mathematical programming and a heuristic algorithm has been developed to solve it. Computational experiments using benchmark instances show that the heuristic algorithm either provides the optimal solution, if available, or the best solution, otherwise. Moreover, the computational times invested are small enough to allow the application of the heuristic in real scenarios.

Keywords: heuristic algorithm, cluster balancing, lexicographic optimization

AMS Classification: 90B06, 90C29, 90C59

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Three-stage Peer methods for the numerical solution of second order IVPs

M. Calvo[1], J. I. Montijano[1], L. Rández[1]

SUMMARY

In this work, we solve numerically second order initial vale problems $y'' = f(t, y)$ by means of 3-stage explicit two-step Peer methods, given by

$$\begin{aligned} Y_{m+1} &= BY_m + hAZ_m + h^2QF_m + h^2RF_{m+1}, \\ Z_{m+1} &= \widehat{B}Z_m + h\widehat{Q}F_m + h\widehat{R}F_{m+1}, \end{aligned} \tag{1}$$

where the stage vectors evaluated at $t_{mi} = t_m + c_i h$ are

$$\begin{aligned} Y_m &= (Y_{mi}), \text{ where } Y_{mi} \simeq y(t_{mi}), \\ Z_m &= (Z_{mi}), \text{ where } Z_{mi} \simeq y'(t_{mi}), \\ F_m &= (f(t_{mi}, Y_{mi})), \end{aligned} \tag{2}$$

and B, A, Q, R, $\widehat{B}$, $\widehat{Q}$, $\widehat{R}$ denote the matrices $s \times s$ of the method, being R, $\widehat{R}$ strictly inferior triangular.

We propose a 3-stage method with one reused stage, so that only two effective function evaluations of the derivative are needed per step. We analyze the 0-stability, consistency and convergence of a particular scheme of order five.

Keywords: Second order equations, Peer methods, stability

AMS Classification: 65L10

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Analyzing with Bayesian models the climate change in series of maximum daily temperature in Aragón

Alejandro Camón[1], Jorge Castillo-Mateo[1], Jesús Asín[1], Ana Carmen Cebrián[1]

SUMMARY

A Bayesian autoregressive model for local daily maximum temperature series, denoted with Y_t for day t, is proposed in the line of Castillo-Mateo et al. [1]. A joint regression model for mean value and variance is proposed for local series, in contrast with the model of these authors, which is spatio-temporal but considers the variance constant over time.

The model must be able to represent the characteristics of the conditional distribution Y_t due the previous temperature, with linear predictors including Y_{t-1}, to express the serial correlation, including harmonic terms cycling in the year, to capture the seasonality, and some trend that could express the evolution in the long term. A Gaussian distribution is considered for residual error.

The full Bayesian statistical inference is solved with Hamiltonian Monte Carlo, a Markov Chain Monte Carlo (MCMC) method that uses the derivatives of the density function being sampled to generate efficient transitions spanning the posterior distribution. This method uses an approximate Hamiltonian dynamics simulation based on numerical integration which is then corrected by performing a Metropolis acceptance step. Libraries of statistical environment R that link with the **'stan'** framework are used for inference.

The model is fitted to a database with 18 daily maximum temperature series located around Aragón and inference results are compared with those of Castillo-Mateo et al. [1].

Keywords: Bayesian model, autoregressive model, variance model, Rstan

AMS Classification: 62F15, 62M10, 62J05

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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A model for stochastic dependence implied by failures among deteriorating components

Emilio Casanova, Sophie Mercier[1], Carmen Sangüesa[2]

SUMMARY

In this work a reliability model in which the failure of components affect to the degradation of components still alive is studied. It is assumed that the initial degradation of each component is described in terms of a non-decreasing univariate Lévy process modified by means of a time-scaling function, and the failure of each component is produced when its degradation crosses a fixed threshold. At the beginning, the components work independently, but once the failure of a component is produced, the time scaling function of the components still alive is modified, in order to include the (possible) stress suffered because of the failure. Probabilistic properties of this model are studied, as well as the reliability of a k-out of n system (or in general, any coherent system).

Keywords: reliability, functional dependency, Lévy processes...

AMS Classification: 62N05, 62H05

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Shape optimization for heat exchangers

Fabien Caubet[1], Carlos Conca[2], Marc Dambrine[1], Rodrigo Zelada[1,2]

SUMMARY

A heat exchanger is a device that allows the heat exchange between two or more fluids without mixing of fluids. The aim of this work is to consider a multi-objective shape optimization in this context, namely to maximize the heat exchange and to minimize the pressure drop.

In [4] it was showed that the cylinder is not optimal for the energy dissipation of the fluid, using the eulerian derivative. In [3] a framework of two fluids separated by the solid distance was considered, imposing a minimum distance between the two fluids, using a lagrangian approach to get the shape derivative. Our work is a continuation of these previous works and a first step towards engineering/industrial applications. Particularly, we aim to take into account the thickness of the material that divides the fluids (that is the pipe). This would require a very fine mesh of this solid region, which is numerically too expensive. Hence, in order to avoid that difficulty, we use asymptotic analysis to obtain an effective transmission condition between the two fluids that takes into account the diffusion in the solid, without meshing it, thanks to the so-called Ventcell conditions.
Concerning the volume model, it is presented as a weak-coupled problem, between the steady-state Navier-Stokes equations for the two fluids dynamics and the convection-diffusion equation for the heat. We characterize the shape derivative for the objective functionals and perform numerical simulations in two and three dimensions to get an optimum heat exchanger, using FreeFem++ as in [2] to implement the numerical algorithm. In particular an objective is to improve the performance of an existing sine-helical heat exchanger presented in [1].

Keywords: Shape optimization, Navier-Stokes equations, heat exchanger.

AMS Classification: 74P10, 76B75, 74F05

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Finite element analysis for a problem with the Ventcel boundary condition

Fabien Caubet[1], Joyce Ghantous[1], Charles Pierre[1]

SUMMARY

Let Ω be an open-bounded and connected domain of $\mathbb{R}^n$ (n=2,3) with $\Gamma = \partial\Omega$ as its compact smooth boundary. We define a finite element method for numerically approximating the solution of the following system:

$$\left\{ \begin{array}{rcll} -\Delta u + \kappa u & = & f & \text{in } \Omega, \\ -\beta\Delta_\Gamma u + \partial_{\rm n} u + \alpha u & = & g & \text{on } \Gamma, \end{array} \right.$$

where n is the outer unit normal vector on Γ, $f \in \mathrm{L}^2(\Omega)$ and $g \in \mathrm{L}^2(\Gamma)$, $\kappa \geq 0$, $\alpha > 0$ and $\beta > 0$ are constants. We discretize the domain Ω and we wish to compare the error between the solution of the exact problem $u \in \mathcal{H} = \mathrm{H}^1(\Omega) \cap \mathrm{H}^1(\Gamma)$ which we equipped with the norm $\|v\|_{\mathcal{H}} = \sqrt{\|v\|^2_{\mathrm{H}^1(\Omega)} + \|v\|^2_{\mathrm{H}^1(\Gamma)}}$ and the solution of the discrete formulation u_h defined on the approximated domain Ω_h. However each function is defined on a different domain, to overcome this problem we will estimate the error between the exact solution and the solution of the lifted problem using the transformation defined in [4], [3] and [2]. Denote u_h^ℓ the lift of u_h on Ω, then our main result is the following error estimate where we use a P^k finite element space ($k \geq 1$):

$$\|u - u_h^\ell\|_{\mathcal{H}} = O(h^k + h^{r+1}),$$

where r is the geometrical degree of approximation of Ω and h is the biggest diameter of a cell of the mesh. Finally we perform numerical simulations which validate this result.

Keywords: Laplace-Beltrami operator, Finite element method, lifted functions, error analysis, geometric error, eigenvalue and eigenvectors approximation.

AMS Classification: 74S05, 65N15, 65N30.

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Some techniques for the stabilization of the pressure discretization in Reduced Order Models of incompressible fluids

Tomás Chacón

SUMMARY

In this talk we address the stability of the pressure discretisation for Reduced Order Models (ROMs) of incompressible flows. For Galerkin discretisations of incompressible fluids, the stability of the pressure is guaranteed through the discrete inf-sup condition for the duality velocity - pressure gradient. This property can be extended to ROMs by adding velocity "supremisers" (the Riesz representation of the pressure gradient on the velocity space, cf. [2]). However it is rather costly and several alternative strategies can be carried on. Among them, stabilisation techniques (cf. [1]) or post-processing of the pressure (cf. [3]), that we shall present in this talk.

Keywords: Reduced Order Modelling, Incompressible flows, Pressure discretisation, Stabilisation

AMS Classification: 65Mxx, 76Dxx

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shapes

Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Existence and regularity of a magnetohydrodynamic system with Navier-type boundary conditions in 2-D.

Amrouche Cherif, Wehbe Elsy[1]

SUMMARY

Magnetohydrodynamic (MHD) is the discipline studying the behaviour of conductive fluids of electricity when their movement is coupled to the electromagnetic field. Here we study in Ω, a multi-connected two dimensional domain, the existence of solutions for a MHD coupling an equation of polymer aqueous solution with Maxwell equation of electromagnetic. These equations are presented, in the stationary case, as the following:

$$\begin{aligned} &-\nu\Delta\boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)(\boldsymbol{u}-\alpha\Delta\boldsymbol{u}) + \nabla\pi - (\boldsymbol{B}\cdot\nabla)\boldsymbol{B} + \frac{1}{2}\nabla(|\boldsymbol{B}|^2) = \boldsymbol{f} && \text{in } \Omega,\\ &-\Delta\boldsymbol{B} - (\boldsymbol{B}\cdot\nabla)\boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{B} + \nabla\theta = 0 && \text{in } \Omega,\\ &\operatorname{div}\boldsymbol{u} = 0, \quad \operatorname{div}\boldsymbol{B} = 0 && \text{in } \Omega, \end{aligned}$$

where $\boldsymbol{u}$ and $\boldsymbol{B}$ are the velocity field and the magnetic field, π is the pressure of the fluid, θ is an unknown function related to the motion of heavy ions and $\boldsymbol{f}$ is the external force acting on the fluid. We study the existence of solutions $(\boldsymbol{u}, \boldsymbol{B}, \pi, \theta)$ in $\boldsymbol{H}^2(\Omega)\times\boldsymbol{H}^2(\Omega)\times\boldsymbol{L}^2(\Omega)\times\boldsymbol{H}^1(\Omega)$ with the Navier-type boundary conditions for $\boldsymbol{u}$ and for $\boldsymbol{B}$. To solve our problem we need some estimations related to the Stokes associated problem. One of the difficulties is the geometry of the domain, supposed here non simply connected. On the other hand, it is shown an additional regularity in $\mathbf{W}^{2,p}(\Omega)$ for the magnetic field.

Keywords: Stokes problem, Magnetohydrodynamic, Navier type boundary conditions, Galerkin method, regularity.

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Optimization models and algorithms for balancing physician shift scheduling in Emergency Departments

Marta Cildoz[1], Pedro Mateo[2], Fermin Mallor[1]

SUMMARY

The shift assignment problem in staff scheduling, even in a restricted real version, is NP-hard [1]. In this presentation, we deal with the Emergency Department (ED) Physician scheduling problem, which is a specially complex problem in this category. It addresses each and every detail of the real-life situation, including the real work calendar and a one-year planning horizon. There are many different types of shifts âĂŞ of varying lengths-, to accommodate a non-uniform daily shift demand dependent on day type, and many ergonomic constraints imposed by mandatory and personnel requirements. The objective function pays attention to the fairness of the schedules among physicians, which entails balancing the distribution of different types of shifts among physicians under a range of often conflicting criteria. Physicians differ in their dedication and the type of shifts they can work.

The problem is initially modelled as an ILP problem but, after a real instance of this problem remains unsolved by a well-known ILP solver in one week, metaheuristic-based algorithms are designed. The first approach considered a GRASP-based algorithm, where the fitness function uses memory elements and the solution of an LP problem, solving a physiciansâĂŹ demand-covering problem. The local improvement combines a Variable Neighborhood Descent Search algorithm and Network Flow Optimization models [2]. The schedules were implemented for four years at the ED of the Hospital of Navarre. Additional constraints and criteria made us develop a matheuristic algorithm that iteratively combines random partial constructive phases with small ILP problems. Solutions obtained by this algorithm outperformed the solutions of the first one, being used at the hospital for the last two years.

Keywords: OR in health services, GRASP, Matheuristic, Physician scheduling Problem

AMS Classification: 90B90, 90C10, 90C59

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Efficient numerical algorithms for solving semilinear singularly perturbed convection-diffusion-reaction systems.

C. Clavero[1], J.C. Jorge[2],

SUMMARY

In this talk we introduce and analyze a technique to develop numerical algorithms for solving a class of semilinear parabolic singularly perturbed systems of convection-diffusion type which have nonlinear reaction terms. We pay special attention to systems where small diffusion parameters with different orders of magnitude are present; this feature provokes that overlapping boundary layers appear in their solutions, close to the outflow boundary. Our proposal combines a linearized version of the fractional implicit Euler method together with a splitting by components, to discretize in time, and the upwind finite difference scheme on appropriate piecewise uniform meshes, to discretize in space. It is proven and checked that the proposed numerical algorithms are uniformly convergent.

Keywords: singular perturbation, uniform convergence, splitting.

AMS Classification: 65M06, 65M12, 65M50

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Fractional models and diffusion type equations

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Mid-point embedding of Hamiltonian systems and variational integrators

J. Cresson and R. Safi

SUMMARY

We discuss the definition of discrete Hamiltonian systems in the context of the mid-point embedding. This particular setting is a first step toward the formulation of discrete Hamiltonian for high-order Galerkin emebeddings. We discuss the mid-point derivative and antiderivative, as well as the associated calculus of variations and we derive the expression of the mid-point Euler-Lagrange equation. We compare our result with the classical approach by J.E. Marsden and coworkers on high order variational integrators.

Keywords: Hamiltonian system, variational integrators, embedding formalism.

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Positive solutions of a slightly subcritical elliptic problem via Orlicz spaces

Mabel Cuesta, Rosa Pardo[2],

SUMMARY

We consider the following elliptic problem

$$\begin{cases} -\Delta u = \lambda u + a(x)\frac{u^{2^*-1}}{[\ln(e+u)]^\alpha}, & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \end{cases}$$

where λ is a real parameter, $\alpha > 0$ is a constant and $a \in C^1(\overline{\Omega})$ changes sign. We use standard variational methods to prove the existence of positive solutions where the main issue is the validity of the Palais-Smale condition. We propose here an Orlicz spaces approach to get the necessary compact embedding.

Keywords: Positive solutions, subcritical nonlinearity, changing sign weight

AMS Classification: 58E07, 35J20, 35B32, 35J25, 35J61

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Least action solution and least action nodal solution for Schrödinger equation on metric graphs

Colette De Coster[1],

SUMMARY

In this talk, we consider the problem

$$\begin{cases} u'' + |u|^{p-2}u = \lambda u, & \text{on the edges of } \mathcal{G} \\ u \text{ continuous and } \displaystyle\sum_{e \succ \mathrm{v}} \frac{du}{dx_e}(\mathrm{v}) = 0, & \text{at the vertex of } \mathcal{G} \end{cases} \tag{1}$$

set on a metric graph $\mathcal{G}$.

The solutions of this problem are the critical points of the action functional

$$J_\lambda(u) := \frac{1}{2}\|u'\|^2_{L^2(\mathcal{G})} + \frac{\lambda}{2}\|u\|^2_{L^2(\mathcal{G})} - \frac{1}{p}\|u\|^p_{L^p(\mathcal{G})},$$

defined on $H^1(\mathcal{G})$.

Two important levels of J_λ are given by

$$c_\lambda(\mathcal{G}) := \inf_{u \in \mathcal{N}_\lambda(\mathcal{G})} J_\lambda(u)$$

where

$$\mathcal{N}_\lambda(\mathcal{G}) := \{u \in H^1(\mathcal{G}) \mid u \neq 0, \mathrm{d}J_\lambda(u)[u] = 0\}$$

and

$$\sigma_\lambda(\mathcal{G}) := \inf_{u \in \mathcal{S}_\lambda(\mathcal{G})} J_\lambda(u),$$

where $\mathcal{S}_\lambda(\mathcal{G})$ is the set of $H^1(\mathcal{G})$ solutions of the problem (1).

In case $c_\lambda(\mathcal{G})$ is attained, it is well known that the corresponding minimum is a solution of (1). In the first part of this talk we will consider the case where $c_\lambda(\mathcal{G})$ is not attained. We can wonder what are the relations between $c_\lambda(\mathcal{G})$ and $\sigma_\lambda(\mathcal{G})$? Are they equal ? Can we have $c_\lambda(\mathcal{G})$ not attained and $\sigma_\lambda(\mathcal{G})$ attained ?

In the second part of the talk, according to the time left, we will consider the problem of existence of sign-changing solutions of (1).

This is based on joint works with Simone Dovetta (Politecnico di Torino), Damien Galant (UMons - UPHF), Enrico Serra (Politecnico di Torino) and Christophe Troestler (UMons).

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On the reconstruction of a function from nonconforming elements by using triangular Shepard basis functions

Francesco Dell'Accio, Filomena Di Tommaso[1], Allal Guessab[2], Federico Nudo[3]

SUMMARY

Most classical numerical methods for approximation of a multivariate function (or integrals of it) use function values at sample points. However, as shown in (cf. [2]), in many practical problems, the available data are not restricted by function evaluations, but contain several integrals over certain hyperplane sections, or, more generally, over smooth surfaces in $\mathbb{R}^d$. In such cases, generalizations of the existing theory and algorithms of approximation operators are required, which are based on the enriched set of data. In this work, we focus on this problem in the two dimensional case, in the setting of scattered data. More precisely we construct new Shepard type approximation operators, based on new enrichments of the standard linear triangular element, using polynomial functions. In line with previously considered improvements of the triangular Shepard method (cf. [3]), these enriched triangular elements will be blended by using triangular Shepard basis functions (cf. [1]).

Keywords: enriched finite element method, nonconforming finite element, triangular Shepard method

AMS Classification: 68W25

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Different Models to solve Non-Hydrostatic Pressure Shallow Flows with Roe-type Riemann Solvers

Isabel Echeverribar[1,2], Pilar Brufau[1], Pilar García-Navarro[1]

SUMMARY

Roe-type Riemann solvers are widely applied to solve the Shallow Water Equations (SWE) in hydrostatic systems. However, when extending these models to include non-hydrostatic effects, other simpler schemes has been frequently used in the past, entailing a big difference between hydrostatic and non-hydrostatic numerical models. This work extends a finite volume numerical scheme previously designed for hydrostatic SW formulation that has been applied with great success in large domains, to a Non Hydrostatic Pressure (NHP) depth averaged model. Additionally, the work explores the available options in the context of previous work in this field: Hyperbolic-Elliptic (HE-NHP) formulations solved with a Pressure-Corrected technique (PCM) [1] and Hyperbolic Relaxation formulations (HR-NHP) [2], in order to find the most suitable model to be solved with a Roe scheme. The performance of both models are compared. The extension of the scheme is assessed obtaining good results. The necessity of understanding the behaviour of the model and the numerical scheme is highlighted.

Keywords: Non-Hydrostatic Pressure, Hyperbolic-Elliptic, Hyperbolic relaxation, Roe Solver

AMS Classification: 35-04, 64-05, 76-04

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Eigenspace Decomposition of Mixed Hodge Structures on Alexander Modules

Eva Elduque[1], Moisés Herradón Cueto[2]

SUMMARY

In previous work jointly with Geske, Herradón Cueto, Maxim and Wang [1], we constructed a mixed Hodge structure (MHS) on the torsion part of Alexander modules, which generalizes the MHS on the cohomology of the Milnor fiber for weighted homogeneous polynomials. The cohomology of a Milnor fiber carries a monodromy action, whose semisimple part is an isomorphism of MHS. The natural question of whether this result still holds for Alexander modules was then posed. In this talk, we will talk about the work in [2] regarding the solution to this question, as well as some consequences and explicit computations.

Keywords: infinite cyclic cover, Alexander module, mixed Hodge structure, formal manifolds

AMS Classification: 14C30, 14D07, 14F45, 32S30, 32S35, 32S40, 55N30

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Alexander Modules and Mellin transform

Eva Elduque[1], **Moisés Herradón Cueto**[2], **Laurenţiu Maxim**[3], **Botong Wang**[4]

SUMMARY

I will talk about the study of Alexander modules of algebraic varieties using Gabber and Loeser's Mellin transform. The main strength of this approach is that it allows the application of the full machinery of the theory of perverse sheaves, and even mixed Hodge modules. We obtain new results about the structure of Alexander modules, especially about their torsion part and, in the multivariable case, their artinian submodules. It also yields a mixed Hodge structure on the maximal artinian submodules of the Alexander modules.

Keywords: Alexander modules, Mellin transform, Hodge structures

AMS Classification: 14C30, 14D07, 14F35, 14F45, 32S35, 32S60, 55N30, 58K15

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Time and space generalized diffusion on networks

Ernesto Estrada[1], **Fernando Diaz-Diaz**

SUMMARY

Normal and anomalous diffusion are ubiquitous in many complex systems. Here, we define a time and space generalized diffusion equation (GDE), which uses fractional-time derivatives and transformed d-path Laplacian operators on graphs/networks. We find analytically the solution of this equation and prove that it covers the regimes of subdiffusion, normal diffusion, and superdiffusion as a function of the two parameters of the model. We extend the GDE to consider a system with temporal alternancy of normal and anomalous diffusion which can be observed for instance in the diffusion of proteins along a DNA chain. We perform computational experiments on a one-dimensional system emulating a linear DNA chain, where we shown that a subdiffusive-superdiffusive alternant regime allows the diffusive particle to explore more slowly small regions of the chain with a faster global exploration, than a subdiffusive-subdiffusive regime. Therefore, an alternancy of sliding (subdiffusive) with hopping and intersegmental transfer (superdiffusive) mechanisms show important advances for protein-DNA interactions.

Keywords: graph theory, fractional calculus, anomalous diffusion, DNA repair

AMS Classification: 05C81, 05C82, 34A08, 60J60

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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The Watson Lemma: a convergent modification

Chelo Ferreira[1], José Luis López[2], Pedro Palacios[2], Pedro Pagola[2], Ester Pérez Sinusía[1]

SUMMARY

Watson's Lemma provides an asymptotic expansion of Laplace transforms for large values of the transformation parameter z. It is a useful tool in the asymptotic approximation of special functions that have an integral representation in the form of the Laplace transform of a certain function $f(t)$. But in most of the important examples of special functions, the asymptotic expansion derived by means of Watson's Lemma is not convergent. We investigate a modification of Watson's Lemma that transforms the unbounded integration interval $[0, \infty)$ of the Laplace transform into the bounded interval $(0, 1]$. Then, we derive an asymptotic expansion of the transformed integral for large z that it is convergent under a mild condition over the function $f(t)$. Moreover, we extend the idea to two dimensions, deriving asymptotic expansions of two-dimensional Laplace transforms for large values of the two transformation parameters that are also convergent. The expansions are accompanied by error bounds. Some examples of special functions are given as illustration, deriving new convergent and asymptotic expansions of these functions.

Keywords: Asymptotic expansions of integrals, Watson's Lemma, Laplace transform, convergent expansions

AMS Classification: 41A60, 41A58, 33F99

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Uniform convergent expansions of integral transforms: Application to special functions

Chelo Ferreira[1], José L. López[2], Ester Pérez Sinusía[1]

SUMMARY

Series expansions of special functions with respect to different systems of functions are interesting representations from an analytical and numerical point of view. Usually, existing expansions for these functions are not simultaneously valid for small and large values of the variables. In this work, we face the problem of designing a general theory of uniform convergent expansions of special functions in terms of elementary functions valid in a large region of the complex plane that includes small and large values of the variables. Error bounds and numerical experiments showing the accuracy of the approximations are given, and its application to important special functions.

Keywords: convergent expansions, uniform expansions, special functions

AMS Classification: 33B20, 33C10, 33C15, 33C75, 41A58, 41A80

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Recent advances in high order numerical methods for fluid dynamics

E. Ferrer[1,2]

SUMMARY

We present the latest developments of our **H**igh-**O**rder **S**pectral **E**lement **S**olver (`HORSES3D`), [1], an open source high-order discontinuous Galerkin framework, capable of solving a variety of flow applications, including compressible flows (with or without shocks), incompressible flows, various RANS and LES turbulence models, particle dynamics, multiphase flows, and aeroacoustics.

Recent developments allow us to simulate challenging multiphysics including turbulent flows, multiphase and moving bodies, using local p-adaption and fast multigrid time advancement. In addition, we also present recent work that couples Machine Learning techniques and high order simulations [2, 3, 4].

Keywords: high order discontinuous Galerkin, machine learning

AMS Classification: 35 (pdes), 65 (num analysis), 76 (fluids)

References

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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A Time–Dependent Generalisation of a Radzievskij Problem

Luis Floría

SUMMARY

V. V. Radzievskij [2] tackled the problem of motion of two point masses inside a homogeneous, rarefied spherical cloud, assuming that the mutual interactions between these point bodies and between the bodies and the particles of the cloud are described by their mutual Newtonian gravitational attraction. As an additional, symplifying hypothesis, the constant density of the cloud is supposed to be sufficiently small so that the resistance of the surrounding material medium to the motion of those point masses might be neglected.

Under these assumptions, *the problem of relative motion of these two bodies can be reformulated as a perturbed Keplerian system, in which the perturbing effects are due to a conservative central force.* Taking advantage of the first integrals of the angular momentum and the energy, Radzievskij followed the conventional solution procedure in plane polar coordinates $(r\,,\,\varphi)$ within the (invariant) plane of motion, formally leading to an orbit equation (in inverted form), $\varphi = \varphi(r\,;r_0\,,\,\varphi_0)$, in terms of *an Abelian integral.* Note that, although this author states that the problem can be analytically solved in terms of Abelian integrals, the fact is that the required quadratures can be reduced to *elliptic integrals.*

In this talk we consider analytical approaches to the solution of *a time–dependent generalisation of the said Radzievskij problem* that can be formulated as the study of motion of a unit–mass particle within a certain time–dependent central–force field.

In particular, the generalized Radzievskij problem under consideration can be viewed as *a perturbed Gyldén system* (that is, a Keplerian system with a time–varying Keplerian coupling parameter $\mu(t)$) *in which the perturbing force is also a time–dependent central force.* This problem does not possesses the classical first integral of the total energy of the system any more, but the force model can be derived from a time–dependent scalar potential.

To deal with this time–dependent central–force problem we take our cue from Deprit ([1], §3, pp. 7–10) and identify the elliptic integrals required to develop the transformation.

Keywords: Celestial Mechanics, Radzievskij's three–body problem, perturbed Gyldén systems, time–dependent central force, canonical transformations, generating functions.

AMS Classification: 70 F 15, 70 F 05, 70 M 20.

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Implicit and semi-implicit high-order well-balanced finite-volume methods for general 1D systems of balance laws

Irene Gómez-Bueno[1], S. Boscarino[2], M.J. Castro[3], C. Parés[3] G. Russo[2]

SUMMARY

The aim of this work is to design implicit and semi-implicit high-order well-balanced numerical methods for general one-dimensional systems of balance laws. The technique introduced by two of the authors in [1] for explicit methods is considered, whose key is the application of well-balanced reconstruction operators. The well-balanced property is preserved when quadrature formulas are used to approximate the cell averages and the integrals of the source term in the cells. This procedure is combined with a time discretization method for the time evolution of type RK-IMEX or RK-implicit (see [2]). The methodology will be applied to several systems of balance laws, ranging from simple scalar problems such as the Burgers equation to more complex systems like the shallow water equations with Manning friction.

Keywords: PDEs,systems of balance laws, finite-volume methods, high-order methods, implicit methods, IMEX methods, well-balanced methods...

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Robust local flux reconstruction for diffusion problems with discontinuous coefficients

Aimene Gouasmi[1], Daniela Capatina[2]

SUMMARY

This work is part of the PhD thesis of A. Gouasmi. One of the goals of the thesis is to reconstruct locally conservative fluxes for finite element approximations of different boundary/interface model problems.

Conservative fluxes are of interest for physicists in many applications such as heat transfer or porous media problems, but they are also used to define simple a posteriori error estimators which are employed in adaptive mesh refinement. Thus, another goal of this work is to carry on a posteriori error analysis based on the numerical fluxes. The literature on flux reconstruction is very rich ; in this paper we follow the approach proposed in [1] for the Poisson problem in a polygonal domain, discretized by standard finite element methods (conforming, nonconforming and discontinuous Galerkin).

This poster is devoted to the extension of the previous paper to the case of a diffusion problem with highly discontinuous coefficients. We only consider a conforming finite element approximation, which is the most difficult to treat, and we focus on the robustness of the reconstruction and of the a posteriori error analysis with respect to both the mesh and the coefficients.

As a perspectives, we will consider a diffusion problem with an interface which does not follow the mesh, and with diffusion coefficients which are discontinuous across the interface. For the numerical treatment of the interface, we employ the CutFEM method [2].

Keywords: flux reconstruction, finite element methods, a posteriori error analysis,...

AMS Classification: 65N12, 65N30, 35J20

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Bilinear control problems associated to chemotaxis models

F. Guillén-González, M.A. Rodríguez-Bellido[1], E. Mallea-Zepeda[2], P. Braz e Silva, C. F. Perusato[3]

SUMMARY

Chemotaxis models try to reproduce the spatial transport of the density of a living organism with respect to a chemical substance. Other interactions between both variables are considered such as production and/or consumption of chemical by cells, degradation of chemical or logistic reaction for living organisms. For several purposes, the control over the system is suitable. In this talk, we consider a bilinear control problem acting on the chemical substance equation and analyze the existence of solution, and the obtention of first-order optimality conditions for local optimal solutions by using a Lagrange multipliers theorem.

Keywords: Chemotaxis model, bilinear optimal control, optimality conditions

AMS Classification: 35K51, 35Q92, 49J20, 49K20

References

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Embedding formalism and high-order variational integrator

Khaled HARIZ-BELGACEM[1,2], Jacky CRESSON[1],

SUMMARY

We provide a general framework of the so-called embedding formalism. We focus on the differential, integral and variational embeddings. Precisely, we study the preservation of variational structures Lagrangian using such framework. Then, we develop a high-order calculus of variations in order to derive the high-order discrete Euler-Lagrange equation. We compare our construction with the *Marsden-West approach* [1].

Keywords: High-order variational integrators, Lagrangian system, Embedding formalism.

AMS Classification: First code, second, third

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Comparing stabilized methods and approaches for the simulation of buoyant flows

Guillermo Hauke[1], Jorge Lanzarote[2]

SUMMARY

Two stabilized methods for the simulation of buoyant low speed flows are presented and compared. The formulations are based on the unified approach for compressible and incompressible flows [1], which solves monolithically the continuity, momentum and total energy equations.

The first strategy uses the Boussinesq approximation to account for the temperature driven forces. This method models the thermal terms in the momentum equation through a temperature-dependent nonlinear source term. The SUPG and SGS stabilized methods will be applied to solve this set of equations. It is known that the Boussinesq approximation poses numerical challenges for high Rayleigh numbers, which manifest in slow or lack of convergence.

The second approach introduces variable density thermodynamics of the liquid or gas without any artificial buoyancy terms, thus, without introducing any approximate models into the full Navier-Stokes equations. Furthermore, this formulation holds for flows driven by high temperature differences and it is thermodynamically consistent.

Various benchmarks [2,3] will be used to illustrate the performance and advantages of each approach. Finally, the technology will be applied to study passive energy efficient devices to climatization.

Keywords: Buoyancy, Boussinesq approach, climatization, stabilized methods, SUPG, SGS, VMS adaptivity

AMS Classification: 65L11,65M60

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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On the minimum of a random number of independent random variables having power function distribution

Pedro Jodrá[1]

SUMMARY

The distribution of the minimum of a random number of random variables having power function distribution defined on the unit interval is established in three cases, in which the resulting models are unit distributions obtained from well-known probability models. To be more precise, let $Z_1, Z_2, \ldots$ be a sequence of independent random variables following a common power function distribution defined on the unit interval. The following characterization results are provided.

(i) Let N be a random variable having geometric distribution which is independent of $Z_1, Z_2, \ldots$ The random variable $T_N = \min\{Z_1, \ldots, Z_N\}$ has unit exponential-geometric distribution (cf. [1, 5]).

(ii) Let M be a random variable having shifted Poisson distribution which is independent of $Z_1, Z_2, \ldots$ The random variable $T_M = \min\{Z_1, \ldots, Z_M\}$ has unit shifted Gompertz distribution (cf. [2, 3]).

(iii) Let W be a random variable having zero-truncated Poisson distribution which is independent of $Z_1, Z_2, \ldots$ The random variable $T_W = \min\{Z_1, \ldots, Z_W\}$ has right truncated Weibull distribution on the unit interval (cf. [4]). Moreover, this result can be extended to an arbitrary interval $(0, c)$ if $Z_1, Z_2, \ldots$ are defined on $(0, c)$, $c > 0$.

Keywords: Minimum of random variables, power function distribution, geometric distribution, Poisson distribution, exponential-geometric distribution, Gompertz distribution, Weibull distribution.

AMS Classification: 60E05, 62F10.

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Fast-slow analysis of dynamical systems and its applications to the study of biological models

Jorge Alberto Jover Galtier[1], Ana Mayora Cebollero[1], Rubén Vigara[1], Sergio Serrano[1], Roberto Barrio[1],

SUMMARY

In the field of dynamical systems, it is common to face problems for which the time scales of their variables are vastly different. These problems are usually tackled by separating the evolutions and characteristics of fast and slow variables. The analysis of the resulting situations helps greatly in the characterisation of dynamics.

In this talk, the basic elements of fast-slow analysis of dynamical systems will be presented, such as singular perturbation theory, critical manifolds and Fenichel's theory. As an application of these tools, the fast-slow analysis of relevant models in the study of biological processes will be presented.

Keywords: Fast-slow analysis, singular perturbation theory, biological models

AMS Classification: 34C60, 70K70, 92B05

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Identification and cure of the checkerboard modes in the solution of both acoustic wave and compressible Euler systems at low Mach number flows

Jonathan Jung[1], Ibtissem Lannabi[1], Vincent Perrier[1]

SUMMARY

It is well known that Godunov type schemes applied to the numerical resolution of the compressible Euler system are not accurate at low Mach number, in particular, on Cartesian meshes. As explained in [1], this inaccuracy is occurring due to a disparity between the continuous and the discrete levels. Over the years, many fixes have been proposed to remedy these deficiencies [2]. However, fixing low Mach number accuracy problem generally introduces other issues like inaccurate acoustic computations, degraded CFL condition and inability to recover the optimal order for a Discontinuous Galerkin discretization of the Euler system [3]. Recently, a new fix was developed in [3] to avoid these problems. However, it still does not escape the checkerboard modes problem that plagues most of other low Mach number fixes.
Interestingly, there is a link between the low Mach number accuracy problem and the long time limit of a linear wave system [4]. This result is useful in explaining the low Mach accuracy problem encountered in the discrete compressible Euler system through an analysis of the linear wave system. In this work, we first propose to exploit this link to identify the origin of the checkerboard modes in the solution of the Euler system. A discrete Hodge-Helmholtz decomposition of the velocity field for the wave system is used to identify the term responsible for the appearance of the checkerboard modes. Next, we investigate the utility of the filtering method towards removing the checkerboard mode from the numerical solution of the linear wave system and eventually the Euler system.

Keywords: Finite volume methods, low Mach flows, checkerboard modes.

AMS Classification: 65M08, 65N22, 76N99

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Modeling of Magnetostriction in Ferromagnetism

Mouna Kassan, Gilles Carbou[1], Mustapha Jazar[2]

SUMMARY

The applications of ferromagnetic materials are more and more numerous: hard disks, recording heads, ferromagnetic paints, nanoelectronics, etc.âĂę We are interested in magnetostriction, ie the influence of mechanical constraints on the magnetization of the ferromagnetic material.
The model, described in [1], is a coupling of the Landau-Lifschitz equation with the elasticity equation. We first establish global existence results in time of the weak solutions of this coupling with mixed boundary conditions for the deformation.
A time averaging method from [2] allows to rigorously obtain a quasi-static model which is a coupling of the Landau-Lifschitz equation with an elliptical equation for the deformation.
Finally, we derive by asymptotic analysis a two-dimensional model of thin ferromagnetic plate which will be more easily calculable than the initial three-dimensional model.

Keywords: Magnetostriction, weak solutions, quasi-stationary model, plates, ...

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On the stability conditions for a heavy gyrostat

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SUMMARY

A gyrostat $\mathcal{G}$ is a mechanical system made of a rigid body $\mathcal{P}$ called the *platform* and other bodies $\mathcal{R}$ called the *rotors*, connected to the platform in such a way that the motion of the rotors does not modify the distribution of mass of the gyrostat $\mathcal{G}$.

In this communication, we will focus on the stability of somne permanent rotations of a heavy gyrostat with a fixed point, that is to say when the gyrostat is under a uniform gravity field. For this case, both necessary and sufficient conditions of stability have been obtained by means of different methods, mainly by using appropriate Lyapunov functions [1, 2, 3, 5]. Provided the system can be regarded as a Lie-Poison one, these results can be obtained and extended by means of the Energy-Casimir method [4]. In this way, we give the stability conditions in terms of the moments of inertia of the gyrostat, the position of the center of mass, as well as in terms of the angular momentum and two gyrostatic moments. Moreover, the necessary conditions are also sufficient, for some configurations of the gyrostat.

Keywords: gyrostat rotation, stability, EnergyâĂŞCasimir method

AMS Classification: 70E55; 37J25; 37N05

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Wave-structures interactions

David Lannes[1]

SUMMARY

The dynamics of floating structures is complex since their motion is governed by the waves, and that this latter is also affected by the presence of the floating structure. The modelling of these interactions raises several mathematical and numerical issues that I will comment on.

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Stokes and Navier-Stokes equations with friction laws at the boundary of the domain and coupling of two fluids

Roger Lewandowski[1]

SUMMARY

In this talk we will review some existence results of weak solutions for the incompressible Navier-Stokes equations with friction conditions at the boundary, such as the Navier law or the nonlinear Glaucker-Manning law. We will then consider the case of the coupling of two fluids governed by the Stokes equations and coupled by a Navier law at the (rigid) interface, such as the ocean and the atmosphere in the laminar regime. We will show that the solutions converge towards a limit satisfying continuity conditions at the interface when the friction coefficient tends to infinity, which is confirmed by a series of simulations, carried out from a numerical scheme for which we are able to prove convergence.

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Insects moving gaits and patterns

Álvaro Lozano[1], Roberto Barrio[2], Sergio Serrano[2], M. Ángeles Martínez[2], Carmen Mayora[2], Rubén Vigara[1]

SUMMARY

The study of the synchronization patterns in biological processes is a growing discipline. Small networks of neurons model central pattern generators (CPG) that control insect locomotion (see [1, 2]).

Here we want to present some of the results we obtained in [3, 4]. Firstly, we study small CPGs (6-neuron model) for insect locomotion where each neuron follows the Hodgkin-Huxley like model of [2], presenting a *roadmap'*with exhaustive information on the dynamical behavior of a single neuron [3], using Spike-counting diagrams and bifurcation analysis.

Then, we analyze the complete system, performing a quasi-Monte-Carlo sweep coupled with an automatic detection techniques. These methods allow us to obtain a complete picture of pattern evolution on the movement gaits of the CPG leading to a global dominance of the tripod gait on the fast movement regime (see [3, 4]) as shown in the gaits of real animals. Using continuation techniques we explain the transitions of different gaits in the current CPG.

Finally we explore other CPGs with similar behaviour.

Keywords: Dynamical systems, CPG, neuron model, bifurcations...

AMS Classification: 34C60, 92B05

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Source term linearization when solving blood vessel flow using approximate solvers

Juan Mairal, Javier Murillo, Pilar Garcia-Navarro[1],

SUMMARY

The flow of blood in elastic vessels such as veins or arteries is difficult to model, specially in the case of the former [1]. It is common to use the Roe solver and its augmented version to solve numerically the hyperbolic conservation laws that govern the flow, in which the solution to the Riemann problem (RP) is given by the eigenvalues of some linearized version of the homogeneous part of the original equations. The inhomogeneous part of the equations accounts for the interactions with the wall, possibly including friction and the stress from sectional variations of the vessel. To deal with them, the Augmented Roe scheme (ARoe) includes the effect the of the source terms over the inner states of the approximate solution given by the homogeneous equations, ensuring equilibrium in well-balanced solutions. However, one disadvantage of the ARoe method is that all the solutions are represented exclusively with shock-type waves, which compromises the resolution of rarefactions, since those can only be represented by a succession of small shock discontinuities. This complication is most prevalent in cases with *transcritical rarefactions*, where the rarefaction fan extends across the sonic point. Harten and Hyman [2] introduced a family of entropy fixes targeted towards solving this defect of the Roe solver based on wave splitting. Indeed, the problematic wave would be separated into two different wave contributions. How to choose these waves gives rise to the different variations of the fix. This work analyzes the application of the wave-splitting method in the case of transcritical rarefactions in venous flow, building up from previous applications to shallow flows and looking at how increasingly complicated methods perform numerically. The goal is to find a compromise between computational cost and precision in order to devise solvers able to simulate blood flows in situations where transcritical rarefactions might occur, such as when liberating a tourniquet.

Keywords: hyperbolic PDEs, transcritical rarefactions, Augmented Roe

AMS Classification: 776H05, 35L04, 76Z05

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Alexander polynomials and characteristic varieties of arrangements

Manousos Manouras

SUMMARY

There have been various definitions of the Alexander invariants of a knot. Following some of these definitions one can generalise them so as to have coefficients twisted by a linear representation. The Alexander type invariants are known to detect non-trivial topological information(genus, hyperbolic volume of a knot etc). The twisted Alexander polynomial was introduced by Wada for knots and has been studied thereafter for more general manifolds as the complement of algebraic curves or line arrangements. We will discuss the relation of the twisted Alexander polynomial of the exterior manifold of a line arrangement and the twisted Alexander polynomial of its boundary manifold.

We will also deal with the characteristic varieties of line arrangements, studied by various authors such as Zariski, Libgober, Artal. The main problem is to understand if the characteristic varieties are combinatorially determined in general. This is known to be true for their âĂİhomogeneous partâĂİ, which corresponds to the resonance variety, as well as for the translated components having dimension at least one, as they are determined by orbifold pencils. This does not work in the same way for the 0-dimensional translated components. Here we present examples such that the characteristic variety has some translated 0-dimensional global component.

AMS Classification: 57M05, 57Q10, 58K65, 14H30, 14B05, 55N33

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Orthogonal Polynomials with respect to Sobolev inner products. An analytic approach with Applications.

Francisco Marcellán

SUMMARY

In this talk we will analyze sequences of orthogonal polynomials with respect to an Sobolev inner product associated with a vector of positive Borel measures supported on the real line. First of all, a historical sketch of motivations for their study will be pointed out. In particular, we will focus the attention on the so-called coherent pairs of measures ([1]). Second, we will emphasize some results concerning constructive methods of Sobolev orthogonal polynomials, Their connection with matrix analysis and operator theory as well as the convergence of Fourier series in terms of coherent pairs will be discussed for different cases of Sobolev inner products. Finally, some applications to the study of Boundary Value Problems for ODEs in the framework of spectral methods will be given (see [3]). For an overview on orthogonal polynomials in Sobolev spaces you can read [2].

Keywords: Orthogonal polynomials, Sobolev inner products, coherent pairs of measures, Fourier series, boundary value problems, spectral methods

AMS Classification: 33C45, 33C47, 42C05

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Torelli problem for logarithmic sheaves

Simone Marchesi[1]

SUMMARY

A very natural question, that rises in the study of logarithmic sheaves, is to which degree the sheaf $\Omega^1_X(\log D)$ determines the divisor D. Whenever the logarithmic sheaf determines unambiguously the original divisor, such a property, called the Torelli property, often allows to give a nice description of the sheaves in the corresponding moduli space. In this talk we will study the Torelli problem for generalized logarithmic sheaves and, if time allows it, for other special families of divisors.

This is a joint work with S. Huh, J. Pons-Llopis and J. Vallès.

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Implicit and explicit integration of the resistance force in non-Newtonian free surface flows

Sergio Martínez-Aranda[1] & Pilar García-Navarro[1]

SUMMARY

In the context of 2D models for free surface flows, depth-averaged rheological models relate the basal shear stress exerted by the bottom surface on the fluid layer to the depth-averaged local flow features. The resistance force play a key role on the mobility of non-Newtonian shallow flows but their numerical integration in 2D frameworks when dealing with large-scale long-term scenarios is still a challenging issue. In this work, two new approaches for the explicit integration of turbulent and visco-plastic resistance terms in 2D numerical models are proposed. These novel approaches, called integral and differential strategies respectively, ensure the integrated resistance force satisfies the rotation invariance property in any mesh topology, allowing upwind computation of the frictional momentum contribution. Benchmark tests are performed in orthogonal, triangle structured and triangle unstructured meshes. The obtained results have been compared with the classical strategy for the upwind computation of the resistance force [1]. Results demonstrate that the alignment of the flow with the mesh main-directions, which has been previously attributed to faults of the Finite Volume (FV) numerical methods and the insufficient mesh refinement, is directly related to the erroneous procedure for including the 2D resistance term into the local flux balance at the cell edges. Furthermore, a novel implicit centered method for the integration of the 2D resistance force has also been derived for the quadratic-type non-linear resistance formulation [2]. This method allows to compute implicitly the flow linear momentum using only current-state flow variables, improving the computational performance respect to the classical iterative models. Implicit and explicit formulations have been compared in terms of accuracy, robustness and computational efficiency. Despite the implicit procedure fails to converge to steady uniform flow states, the novel explicit upwind and the implicit centered methods show similar level of accuracy, robustness and computational efficiency for transient 2D frictional visco-plastic flows.

Keywords: Geophysical shallow flows, Resistance force integration, Rheological models, Rotational invariance

AMS Classification: 76-10, 76A05, 76M12

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Connecting Chaos in Two-Coupled Brusselators Model

Ana Mayora-Cebollero, Roberto Barrio, Jorge A. Jover-Galtier, Carmen Mayora-Cebollero, Sergio Serrano[1], Álvaro Lozano, Rubén Vigara[2], Fátima Drubi, Santiago Ibáñez, Lucía Pérez[3]

SUMMARY

The Brusselator model (I. Prigogine and R. Lefever, 1968) is a theoretical model that represents an autocatalytic chemical reaction with oscillations (Belousov-Zhabotinsky reaction). If we couple two identical Brusselators by diffusion, we obtain the system that concerns us: the two-coupled Brusselators model. This system has four variables (x_1, y_1, x_2, y_2) and four parameters (A, B, λ_1, λ_2) where λ_1 and λ_2 correspond to the coupling by diffusion.

This coupled system presents different dynamical regimes. For example, in the literature two different chaotic regions [1, 2] have been located in the parameter space. The first parametric region with chaotic behaviour [1] is much larger than the other one [2], which is associated with the existence of Shil'nikov homoclinic orbits. We define a new parameter α to join the biparametric plane (B, λ_1) of both regions. We study in detail the parameter phase space and how all the different elements are connected. In particular, we use several numerical and analytical techniques such as spike-counting sweeping, Lyapunov Exponents, continuation methods and Shil'nikov theory in order to study this coupled system [3].

Keywords: two-coupled Brusselators model, bifurcations, chaos, Shil'nikov theory

AMS Classification: 37Gxx, 37Mxx

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Deep Learning for Chaos Detection

Carmen Mayora-Cebollero, Roberto Barrio,
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Rubén Vigara[2], Antonio Miguel, Alfonso Ortega[3]

SUMMARY

Chaos detection can be considered one of the most important problems that we have to take into account if we want to study the behaviour of a dynamical system. Although it can be solved using classical techniques as Lyapunov Exponents (LEs) [1], recently some authors have proposed to apply Deep Learning [2].

Deep Learning (DL) is the branch of Machine Learning that uses Artificial Neural Networks (architectures based on multiple layers) to learn from data with several levels of abstraction. Researchers have obtained pretty good results using DL to perform non-mathematical tasks, so it is natural to try to apply it in mathematical problems (and in particular, dynamical systems problems). In this talk, we introduce some state-of-the-art techniques that we have used for chaos detection, we show the obtained results and we compare them to the ones that we can obtain with classical techniques as LEs [3].

Keywords: Dynamical Systems, Deep Learning, Chaos Detection

AMS Classification: 37Mxx, 68T07

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A general multivariate lifetime model with a multivariate additive process as conditional hazard rate increment process

Sophie MERCIER, Carmen SANGÜESA[1]

SUMMARY

The object of the present paper is the study of the joint lifetime of d components subject to a common stressful external environment. Out of the stressing environment, the components are independent and the lifetime of each component is characterized by its failure (hazard) rate function. The impact of the external environment is modelled through an increase in the individual failure rates of the components. The failure rate increments due to the environment increase over time and they are dependent among components. The evolution of the joint failure rate increments is modelled by a non negative multivariate additive process, which include Lévy processes and non-homogeneous compound Poisson processes, hence encompassing several models from the previous literature. A full form expression is provided for the multivariate survival function with respect to the intensity measure of a general additive process, using the construction of an additive process from a Poisson random measure (or Poisson point process). The results are next specialized to Lévy processes and other additive processes (time-scaled Lévy processes, extended Lévy processes and shock models), thus providing simple and easily computable expressions. All results are provided under the assumption that the additive process has bounded variations, but it is possible to relax this assumption by means of approximation procedures, as is shown for the last model of this paper.

Keywords: Reliability, Multivariate survival function, Multivariate covariate process, Multivariate additive process, Multivariate Lévy process, Poisson random measure

AMS Classification: 60G51, 60G55, 62H05

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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The use of fractional calculus in functional analysis

Pedro J. Miana

SUMMARY

In this summary talk, we present some use of fractional calculus in different context of functional analysis. In [7], we present how Weyl fractional calculus is connected with α-times integrated semigroups. This study is extended to k-convoluted semigroups in [5]. Discrete version of this treatment are followed in [1] to apply to (C_α)-bounded operators and for discrete Cesàro bounded operators in [2].

The extension problem and fractional powers of generators of α-times integrated semigroup is studied in [3]. Fractional powers of finite difference operators in $\ell^p(\mathbb{Z})$ is presented in [4] using Banach algebras. Finally we extend this work in $\ell^p(\mathbb{N}_0)$ in [6].

Keywords: Weyl fractional calculus, integrated semigroup, finite difference equations

AMS Classification: 26A33, 47D06, 47D62

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Coupling multicomponent geochemical reactions to flow and transport codes

Sergi Molins[1]

SUMMARY

Multi-component reactive transport modeling (RTM) has proven to be a valuable tool to advance mechanistic understanding of hydro-biogeochemical processes in a variety of natural and engineered systems. As the role of RTM expands, new code must sometimes be developed. However, the number of diversity of models used for reactive processes and related mathematical formulations makes the development of new general geochemical capabilities impractical. For this reason, this development is often circumvented by coupling existing geochemical codes to newly developed or updated codes that simulate flow, transport and other processes. The most common example of this is the use of PHREEQc as geochemical engine in many RTM applications.

In this talk, I will present examples of coupling geochemical reactions to flow and transport codes for different problems, from the pore scale to the watershed scale, that illustrate this software development approach. In all cases, the Operator Splitting (OS) approach is used where transport and reactions are solved separately and sequentially over a time step. This makes it possible to solve the geochemical problem in a point-by-point manner and facilitates the parallelization of the code. In two of the cases, rather than using a custom coupling between codes, a generic interface is developed that facilitates interoperability and code re-use. The geochemical capabilities of two existing codes, CrunchFlow and PFLOTRAN, are made available via this interface. Future directions in the development of this interface will be discussed.

Keywords: Pore-scale modeling, reactive flow, porous media

AMS Classification: 76-10, 76S05

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Pore scale modeling of reactive flow and transport processes in evolving porous media

Sergi Molins[1]

SUMMARY

The use of subsurface environments in applications related to energy storage and security often involves the injection of fluids that are far from equilibrium with respect to the constituents of the native formation. Such disequilibrium drives geochemical reactions that change the structure of the porous media when primary minerals dissolve and secondary minerals precipitate. This evolution of the medium in turn affects how fluids move in the subsurface leading to emergent behavior that cannot be easily predicted, with both the development of fast flow pathways and the self-sealing of the medium being possible outcomes. The success or failure of the application is thus determined by this non-linear interaction between reactive flow and transport processes.

Historically, the subsurface has been treated as a porous continuum characterized by properties that are applicable over a representative elementary volume that includes both pores and grains. However, the processes that take place at the scale of individual pores and grains may translate into significant impacts at larger scales via nonlinear emergent processes. Over the last decade and a half, characterization and modeling tools applicable at the pore scale have been developed to understand these impacts.

In this talk, I will describe the development of a pore scale model that has been used over the years to understand how heterogeneous pore structures affect effective reaction rates observed at the porous-continuum scale and how the porous media evolves as a result. The model is based on an explicit representation of the interface between solid and fluid. This makes it possible to capture the transport limitations to the rates computed on reactive surfaces. The governing equations are discretized directly on a Cartesian grid using an embedded-boundary (EB) finite volume method using high-order methods. The code solves separately flow, reactive transport, and boundary displacement over a given same time step, assuming that flow and reactive transport solutions change instantaneously as the geometry evolves.

I will illustrate the use of the model with a number of applications, with an emphasis on those that relied on the use of image data obtained from experiments. I will use these examples to discuss the strengths and weaknesses of the pore scale approach in general as well as those associated with this model in particular. In this context, I will contrast it with other models using results from a benchmark problem that was published in 2019. I will conclude by describing ongoing work to address challenges related to capturing physical and mineralogical heterogeneity at a broad range of spatial scales with pore scale and multiscale models, and by identifying future directions in pore scale modeling.

Keywords: Pore-scale modeling, reactive flow, porous media

AMS Classification: 76-10, 76S05

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Bernstein-Sato polynomials and related invariants for meromorphic functions.

Josep Álvarez Montaner[1], Manuel González Villa[2], Edwin León-Cardenal[3], Luis Núñez-Betancourt[2]

SUMMARY

We describe a family of Bernstein-Sato polynomials for meromorphic functions, and use them to prove some functional equations as well as some generalizations of classical results by Kashiwara and Lichtin. We focus mostly on the use of these Bernstein-Sato polynomials for studying some aspects of the singularities of meromorphic functions like Archimedean zeta functions and multiplier ideals.

Keywords: Bernstein-Sato polynomial, Zeta Functions, Singularities

AMS Classification: 14F10, 32S40, 42B20, 14F18

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Very high order approximation of the Euler equations in presence of buoyant forces

Adrián Navas-Montilla[1], Isabel Echeverribar[2]

SUMMARY

The Euler equations are used to model astrophysical and atmospheric phenomena, in particular for climate modeling and weather forecasting purposes. Most of the phenomena of interest can be regarded as small perturbations of the underlying equilibrium state of relevance [1], called hydrostatic equilibrium, where the the pressure forces are balanced by the gravity force. Capturing such perturbations is a challenging task and require the use of well-balanced schemes, which are able to preserve the hydrostatic equilibrium at the discrete level with machine accuracy. In this work, we introduce a novel approach to construct well-balanced finite volume schemes of arbitrary order of accuracy based on augmented Riemann solvers. We use the HLLS solver to compute the fluxes at cell interfaces, ensuring an exact balance between fluxes and sources at cell interfaces. To the knowledge of the authors, this solver has not been applied to the Euler equations yet. We also introduce a particular discretization of the source term to satisfy the well-balanced property. The high order of accuracy is obtained by means of a Weigthed Essentially Non-Oscillatory (WENO) spatial reconstruction [2] and a Runge-Kutta time integrator. The resulting scheme is assessed using a variety of test cases involving steady and transient solutions. The fidelity of the model for the resolution of vortical structures and the representation of the statistical properties of turbulent flows is also evaluated. As a future work, the model will include the presence of uneven topography using immersed boundaries and will be coupled with a level-set equation solver for the simulation of surface fire.

Keywords: fluid mechanics, finite volumes, Riemann solvers, well-balanced

AMS Classification: 35Q31, 65M08

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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High-Order Spectral Difference Methods for Magnetohydrodynamics

Jonatan Núñez-de la Rosa[1]

SUMMARY

A high-order spectral difference method (SDM) for solving the magnetohydrodynamics equations is presented. Because of the presence of shocks in astrophysical scenarios, the numerical framework consists of a hybrid scheme, where for smooth parts of the flow, the SDM is used, and those regions with strong shocks are evolved with a robust finite volume (FV) method with WENO3 reconstruction. In this approach, we interpret the nodal SDM values in the troubled element as FV subcell values for their further time evolution. The generalized Lagrange multiplier method is employed to enforce the solenoidal constraint on the magnetic field. For the time discretization, an explicit fourth-order strong stability-preserving Runge–Kutta method is employed. Numerical results with very high polynomial degree include the Orszag–Tang vortex, the spherical blast wave problem and the Kelvin–Helmholtz instability. Finally, we also compare this scheme with high-order finite difference, finite volume, and discontinuous Galerkin schemes, analyzing their robustness and performance.

Keywords: Spectral difference methods, magnetohydrodynamics, shock capturing

AMS Classification: 65M70, 76M22, 76W05

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Scaling property for bisectorial operators and an application to averaged Black-Scholes equation

J. Oliva-Maza[1], **M. Warma**[2]

SUMMARY

Sectorial operators play a central role in the theory of abstract homogeneous equations. They satisfy the so-called 'scaling property', i.e. if A is a sectorial operator of angle $\delta \in [0, \pi)$, then A^α is a sectorial operator of angle $\alpha\delta$ for $\alpha \in [0, \pi/\delta)$. In this work, we extend this property so it covers more general functions, in particular those functions whose absolute value has fractional power-like behaviour in some sense. This result is then used to prove the well-posedness of a family of averaged Black-Scholes equations, which involve the Riemann-Liouville and Weyl fractional derivatives.

Keywords: sectorial operators, functional calculus, generalized Black-Scholes equation, fractional derivatives

AMS Classification: 26A33, 47A60, 47B12

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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q-Bernstein bases and q-Bézier curves

Héctor Orera[1], Jorge Delgado[1], Juan Manuel Peña[1],

SUMMARY

The q-Bernstein basis of univariate polynomials, $0 < q \leq 1$, was introduced by Phillips in [2]. This basis has played an important role in several fields such as Computer Aided Geometric Design (CAGD), Approximation Theory or Quantum Calculus and they have received a lot of attention in recent research (see [1, 3], and references therein). This basis contains the classical basis of Bernstein polynomials for the particular case $q = 1$.

In this talk, we will review some important properties of q-Bézier curves and of q-Bézier polynomials. Evaluation algorithms will be presented and an extension for the design of surfaces will also be introduced.

Keywords: q-Bernstein; q-Bézier; corner cutting algorithm

AMS Classification: 41A10, 65D17

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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System for the urgent delivery of face shields during the first wave of the COVID-19 pandemic: use of tabú search and multi-start framework

Joaquín Pacheco[1]

SUMMARY

The speed by which the COVID-19 pandemic spread throughout the world caught some national and local governments unprepared. Healthcare systems found themselves struggling to increase capacity and procure key supplies, such as personal protective equipment. Protective face shields became essential for healthcare professionals. However, most hospitals and healthcare facilities did not have them in adequate quantities. The urgency of producing and delivering face shields increased as the number of COVID-19 cases rapidly multiplied. This was the situation that we encountered in the city and province of Burgos (Spain). Since there was no time to wait for a large manufacturer to produce face shields, private citizens and small companies volunteered to make them using technologies such as 3D printers. Nonprofits, citizens, and governments agencies volunteered to deliver materials to the face shield makers and to pick up and deliver the face shields to health centers and other locations where they were needed. This resulted in a vehicle routing problem with some special characteristics that made it different from models used for commercial purposes. We describe the development of a heuristic to find feasible and efficient routes for this problem. We highlight the advantages of using heuristics in an emergency context like the one triggered by the COVID-19 pandemic. In particular, the heuristic approach allowed us to design, implement, test, and delivery a routing system in less than one week from the time that the local government contacted us with what they described as a logistics nightmare.

Keywords: Humanitarian logistics, Heuristic optimization, Tabu search

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Goodness-of-fit test for homogeneous gamma process under a general sampling scheme

Ch. Paroissin[1],

SUMMARY

Degradation models are more and more used in reliability analysis. Among the most classical models, one can cite the Wiener process with a linear drift and the gamma process. Such models can be used to define complex maintenance policies, for instance. However, up to now, no goodness-of-fit test has been proposed formally for a general sampling scheme. The aim of this paper is to develop such a test for the homogeneous gamma process.

Keywords: Asymptotic normality, Degradation model, Reliability analysis

AMS Classification: 62P30, 62N05

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Some approximation problems by generalized Wendland compactly supported radial basis functions

M. Pasadas[1], P. González[1], A. Hananel[2]

SUMMARY

In a wide range of applications, positive kernels have proven to be very useful tools due to desirable properties such as naturally arise, reproducing kernel Hilbert space of Hilbert spaces of continuous functions, frequently appearing as radial basis functions

$$K(x,y) = \phi(\langle x-y\rangle_d), \quad \forall\, x,y \in \mathbb{R}^d,$$

where $\langle\,\cdot\,\rangle_d$ represents the Euclidean norm in $\mathbb{R}^d$, being $\phi:\,[0,+\infty) \to \mathbb{R}$ a smooth univariant function.

Usually, these kernels are not compactly supported or they are not differentiable. The Wendland functions have no such drawbacks[2]. They are polynomials in $[0,1]$ of minimal degree $\lfloor d/2\rfloor + 3k + 1$ and yield positive definite C^{2k} radial basis functions on $\mathbb{R}^d$. Moreover, they are reproducing kernels of Hilbert spaces isomorphic to the Sobolev space $H^{d/2+k+1/2}(\mathbb{R}^d)$. But this means that integer-order Sobolev spaces in even dimensions are not covered. Thus, it is necessary to extend the classical Wendland functions to the generalized Wendland functions[1] that allow to reproduct kernels of Hilbert spaces isomorphic to integer-order Sobolev spaces in even dimensions while holding the compact support property.

In this work we deal with interpolation and smoothing problems in a finite-dimensional generalized Wendland functions space; we prove the corresponding convergence results showing some graphical and numerical examples in $\mathbb{R}^2$.

Finally, we present a numerical method for solving 2D Fredholm integral equations of second kind by generalized Wendland radial basis functions.

Keywords: Generalized Wendland functions, interpolation, smoothing, 2D Fredholm integral equations

AMS Classification: 65D12, 65D05, 45B05, 65R20

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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A new stabilization based on the mass matrix for Biot's consolidation model

Álvaro Pé de la Riva[1], Carmen Rodrigo[2], Francisco J. Gaspar[3]

SUMMARY

A stabilization is usually required in order to avoid nonphysical oscillations in the numerical solution of the fluid pressure for the quasi-static Biot's model for poroelasticity [1]. For this purpose, we propose to add a mass term as stabilization such that the spurious oscillations are completely eliminated. In addition, this stabilization can be applied in a natural way together with the well-known fixed-stress method [2] for solving the poroelasticy problem. In this work, we will consider pairs of isogeometric elements for discretization. Specifically, we propose the pairs of isogeometric elements $Q1-Q1$, $Q2-Q1$ and $Q3-Q2$ (with global $\mathcal{C}^1$ smoothness). By means of the standard von Neumann analysis, we prove the stability and convergence of these schemes. Finally some numerical results will be shown.

Keywords: Poroelasticity, Biot's model, stabilization, von Neumann analysis, isogeometric analysis

AMS Classification: 65F10, 65L20

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Simulation of reactive flows on 3D pore-scale geometries using particle methods, and their impact on macroscopic properties: applications to dissolution, precipitation and crystallization.

Sarah Perez[1], Jean-Matthieu Etancelin[1], Philippe Poncet[1].

SUMMARY

The study of reactive flows within a porous medium is an essential step in the comprehension and management of the geochemical effects arising in CO2 capture and storage.

While the continuum approach at the Darcy scale finds its effectiveness in modeling the behaviour of natural reservoirs, simulations on pore-scale geometries provide significant insights into the fluid-mineral interfaces and the monitoring of their evolutions due to complex chemical reactions. Through homogenization theory and numerical modeling, we are able to simulate the effects of reactive processes occurring at this pore-scale and predict the impact on the medium main effective properties at the macroscale. The wide range of pore-scale modeling approaches includes, among others, discretization methods, pore network models, lattice Boltzmann method, and particle methods.

This talk will focus on the latter category, and present the advances in the semi-Lagrangian method involving a particle treatment of the chemical species transport coupled with underlying grids for the hydrodynamic [1]. The numerical method developed shows remarkable agreement with the others pore-scale modeling methodologies but also with experimental results, providing cross-validation in a calcite dissolution benchmark [2].

We will also present simulations of precipitation and crystallization in 3D porous media with a two-scale porosity. It includes nucleation of the dissolved chemical species and crystal growth due to their interactions with the solid interfaces. A Particle-Strength-Exchange method [3] is dedicated to the approximation of Archie's law term with heterogeneous diffusion coefficients, and the numerical method is extended to GPU-CPU computing using HySoP HPC platform developed at UPPA-LJK-CNAM.

Keywords: Pore-scale modeling, reactive flow, Darcy-Brinkman, porous media

AMS Classification: 76-10, 76S05, 65M22

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Optimal observation of acoustic waves

Yannick Privat[1]

SUMMARY

We are interested in the optimal properties of some inverse problems involving the wave equation

$$\partial_{tt} p(t,x) - \Delta p(t,x) = 0 \quad (t,x) \in [0,T] \times \Omega,$$

where p denotes an acoustic pressure propagating in a (bounded) cavity Ω and $T > 0$ is the duration of the experiment. Such a model is used for example in thermo/photo-acoustic and ultrasound tomography. A classical inverse problem is the following: *given a measurement (made by sensors) over a certain time T of the pressure p in a subdomain ω of Ω or on a part of $\partial\Omega$, can we reconstruct the initial pressure and wave velocity (at time $t = 0$) in Ω?*

The conditions guaranteeing the well-posedness of this problem are well known and imply a functional inequality called *observability inequality*. This presentation is dedicated to the following problem:

> *Is there an optimal way to position the set of sensors in order to reconstruct the initial data?*

This is a shape optimization problem in which the unknown is the domain occupied by the sensors. I will build on the series of works [1, 2, 3, 5] and will present some concrete applications and more recent results, related to the question : *can we hear the shape of a room?*

This work is in collaboration with Antoine Deleforge (Inria), Cédric Foy (Cerema), Emmanuel Humbert (univ. Tours), Tom Sprunck (Inria and univ. Strasbourg), Emmanuel TrÃľlat (Sorbonne univ.), Enrique Zuazua (Fau. Erlangen)

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Bayesian assessment of a new tumor-growth mathematical model under the action of electrochemical therapy

Erick Eduardo Ramirez-Torres[1], Antonio Selva Castañeda, Juan Ignacio Montijano[2]

SUMMARY

To our knowledge, only a single mathematical modeling approach to the action of electrochemical therapy (EChT) in a malignant tumor has been reported (cf. [1]). This approach assumes that EChT affects tumor growth rate. Here we propose a new modeling approach that assumes the action of electrochemical therapy as an aggressive agent external to the tumor, similar to how chemotherapy is modeled (cf. [2]). The proposed model is evaluated with Bayesian techniques. The posterior distributions of its parameters are sampled with the Monte Carlo delayed rejection adaptive metropolis method (cf. [3]). Both models are evaluated with data from an Ehrlich tumor treated with EChT. The new model shows better values of the Bayesian information criterion (cf. [4]) and outperforms the previously reported model in the value of the fractional Bayes factor (cf. [5]).

Keywords: Bayesian inference, ODE models, tumor growth.

AMS Classification: 62F15, 34A55, 91G60

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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A finite volume method to estimate heating in additive manufacturing parts with magnetocaloric particles

I. Royo-Silvestre[1], **J.C. Jorge**[1],

SUMMARY

The development of additive manufacturing has an increasing impact in industry as a competitive technology that can reduce costs, manufacturing times and delivery times for complex parts. Recently, an innovative research is developing focused at 3D-printing of functional parts, e.g. printing of electronic circuits and magnetic wires. In this direction, collaborating with NAITEC and the Physics department of UPNA, we have designed, implemented and validated a numerical algorithm of type finite volumes to simulate the evolution of temperature in heterogeneous solid rectangular prisms featuring electric or magnetocaloric components. Numeric results have been used to improve part designs and have been compared with experimental data. Finally, numerical orders of convergence of the implemented algorithm will be displayed.

Keywords: finite volumes, magnetocaloric, additive manufacturing

AMS Classification: 65M08, 65Z05, 80M12

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Mathematical model for fitting multiple epidemic waves of COVID-19 in the Aragon community

Antonio Rafael Selva Castañeda [1], Erick Ramirez Torres, Luis Rández, Juan Ignacio Montijano Torcal[2],

SUMMARY

In this study it is proposed a modified Susceptible-Exposed-Infectious-Removed (SEIR) model (cf. [1]) describes the time behavior of symptomatic, asymptomatic, and hospitalized patients in multiples waves of an epidemic of COVID-19 in Aragon community, taking into account the effect of the demographic evolution. Unlike most of the recent studies where a constant ratio of new individuals is considered, we consider a more correct assumption that the growth ratio is proportional to the total population (cf. [2]). This work aims to propose a model that permits the study of the multiple community outbreaks of COVID-19, using for this purpose a deterministic mathematical model combined with surveillance data to obtain a model capable of describing the multiple waves of contagion observed. The model is fitted to experimental data corresponding to the pandemic evolution in Aragon, showing a proper behavior of infected, accumulated, and recovered patients for 6 epidemic waves. In conclusion, the model seems to be an adequate tool for the study and control of infectious diseases in particular the COVID-19 disease transmission.

Keywords: Mathematical model, COVID-19, disease transmission,...

AMS Classification: 92B05, 37N25

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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POD-based ROM modified to predict solutions in time

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SUMMARY

Reduced-order models (ROMs) based on the proper orthogonal decomposition (POD) are widely used to reduce computational costs when compared to standard numerical methods, also called full-order models (FOMs). The ROM strategy consists of two parts: the off-line part, in which the ROM is trained by applying the snapshot method (cf. [1]) to the solutions of the FOM computed up to the training time; and the on-line part, in which the ROM is solved up to the same training time. One of the most important limitations of POD-based ROMs is the prediction of solutions beyond the training time when considering hyperbolic problems. In this work, a modified ROM based on a coordinate transformation (CT-ROM, cf. [1, 3]) is presented which allows to compute the solution of advection-dominated problems beyond the training time. The CT-ROM is tested by applying it to problems of different nature, including linear and non-linear equations and systems of equations. In addition, this strategy is also applied to 2D problems by means of the Radon transform.

Keywords: Reduced-order modelling, POD methods, snapshots method, computational resources, time extrapolation

AMS Classification: 65M08, 35Q35

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Sixteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Fractional models in Biology

Anna Szafrańska[1] Jacky Cresson[2]

SUMMARY

Modelling of natural phenomena like e.g. spread of the virus or interactions between species, using mathematical tools is highly desirable due to the possibility of understanding and analysing the problem. A well-constructed model can be used to observe and predict the dynamics of a given phenomenon. The usual tools based on ordinary or partial differential equations in some cases can be insufficient to capture well the dynamic behaviour, especially in the modelling phenomena where we are dealing with the memory effect. To deal with this problem we can successfully reach for the fractional calculus and take advantage of the non-locality property of the fractional order derivatives.

In this talk we give some examples of superiority of mathematical modelling using fractional calculus over the classical approach. Mostly we focus on the modelling problem of the spread of the dengue fever epidemic, i.e. fractional model construction, numerical approach for model validation, parameters estimation.

Keywords: Fractional calculus, Non-standard numerical methods, Parameter estimation

AMS Classification: 26A33, 49M25, 65Q30

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Optimal control of two dimensional third grade fluids

Yassine Tahraoui[1], Fernanda Cipriano[2]

SUMMARY

We study the optimal control of the velocity field y of an incompressible third grade fluid filling a two-dimensional bounded domain with a smooth boundary. More precisely, our aim is to solve the following problem

$$\min_{U\in\mathcal{U}_{ad}}\{\frac{1}{2}\int_0^T \|y-y_d\|_2^2 dt+\frac{\lambda}{2}\int_0^T \|U\|_2^2 dt\},$$

where the evolution equation of y is given by

$$\partial_t(v(y))-\nu\Delta y+(y\cdot\nabla)v(y)+\sum_{j=1}^{2} v(y)^j\nabla y^j-(\alpha_1+\alpha_2)div(A^2)-\beta div(|A|^2A)=-\nabla\mathbf{P}+U, \quad (1)$$

where $v(y):=y-\alpha_1\Delta y$, $A:=A(y)=\nabla y+\nabla y^T$ and the control acts through the external force U. The constant ν represents the fluid viscosity, α_1,α_2, β are the material moduli, and $\mathbf{P}$ denotes the pressure. The equation (1) will be supplemeted with a homogeneous Navier-slip boundary condition. We recall that $y_d\in L^2(D\times(0,T))$ corresponds to a desired target field and $\lambda\geq 0$. We prove the existence of an optimal solution and establish the first order optimality conditions. Furthermore, an uniqueness result of the coupled system constituted by the state equation, the adjoint equation and the first order optimality condition is established, under sufficiently large intensity of the cost λ.

Keywords: Third grade fluids, optimal control, Necessary optimality condition.

AMS Classification: 35Q35,49K20,76A05, 76D55

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New examples of free projective curves

Jean Vallès[1]

SUMMARY

A projective plane curve is called free when its module of tangent derivations is a free module (generated globally by 2 derivations). This module is a fundamental object intervening in many branches of mathematics but yet still poorly understood. The link between the freeness of this module and the singularities of the curve is the subject of numerous and exciting conjectures, in particular Artal-CogolludoâĂŹs conjecture about pencil of curves.

In 2015, I proved that the union of the singular curves of a generic pencil is free as suggested by the Artal-Cogolludo conjecture. Generalizing this result to any kind of pencils, I will give a necessary and sufficient condition for a union of component of curves of the pencil to be free. This leads to new examples of free curves.

This is a joint work with R. Di Gennaro, G. Ilardi, R. Miro-Roig and H. Schenck.

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Analytic Solutions and Complete Markets for the Heston Model with Stochastic Volatility

Peter Takáč[1], Bénédicte Alziary[2]

SUMMARY

We begin by a brief presentation of a well-known mathematical model for European option pricing in a market with stochastic volatility: the popular *Heston volatility model* (Rev. Financial Studies, 1993). European options are used for *market completion*. We explain the connection between a complete market and the analyticity of the weak solution to a general, strongly parabolic linear Cauchy problem of second order in $\mathbb{R}^N \times (0,T)$ ($N = 2$) with analytic coefficients (in space and time variables). The analytic smoothing property is expressed in terms of holomorphic continuation of global (weak) L^2-type solutions to the system. Given $0 < \xi' < \infty$ and $0 < T' < T < \infty$, we sketch a proof that any L^2-type solution $u\colon \mathbb{R}^1 \times (0,\infty) \times (0,T) \subset \mathbb{R}^2 \times (0,T) \to \mathbb{R}^1$, $u \equiv u(x,v,t)$, possesses a bounded holomorphic continuation $u(x+\mathrm{i}y, \xi+\mathrm{i}\eta, \sigma+\mathrm{i}\tau)$ into a complex domain in $\mathbb{C}^N \times \mathbb{C}$ ($N = 2$) defined by $(x,\xi,\sigma) \in \mathbb{R}^1 \times (\xi',\infty) \times (T',T)$, $|y| < A_1'$, $|y| < A_2'$, and $|\tau| < B'$, where $A_1', A_2', B' > 0$ are constants depending upon ξ' and T'. The proof uses the extension of a solution to an L^2-type solution in a complex domain in $\mathbb{C}^2 \times \mathbb{C}$, such that this extension satisfies the Cauchy-Riemann equations. The holomorphic extension is thus obtained in a (weighted) Hardy space H^2. A serious difficulty in the Heston model is that the solution is sought only in a half-space $\mathbb{H} = \mathbb{R}^1 \times (0,\infty)$ in $\mathbb{R}^2$ with rather complicated dynamic boundary conditions at the boundary $\partial\mathbb{H} = \mathbb{R}^1 \times \{0\}$; a similarity with the Feller boundary condition (Ann. Math., 1951) will be discussed. We avoid this trouble by a suitable choice of the weight in the weighted L^2 space.

Keywords: Space-time analyticity, parabolic PDE, holomorphic continuation, market completeness, European option

AMS Classification: 35B65, 35K10, 91B28

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An adjoint-based optimal control model for unsteady free surface flows

P. Vallés[1], **I. Echeverribar**[2,3], **P. García-Navarro**[2]

SUMMARY

As extreme natural disasters, flood events raise governments, institutions and general society concern because of their destructive effects. The adjustment and control of hydraulic structures is a daily action to obtain certain level and flow values in order to reduce flood damage used to regulate water volumes to downstream regions, as well as to meet the demands of agriculture, livestock and hydroelectric energy production. The temporal and spatial evolution of flood waves in rivers is often simulated under the shallow water hypothesis [1]. This allows the approximation in cross-section, averaged one-dimensional models [2]. At the same time, hydraulic structures such as a reservoir can be modelled using an aggregated formulation, which, avoiding the discretization of the domain and the calculation of some hydraulic variables, contributes to computational efficiency [3]. The main objective of the this work is the development and implementation of a control method based on adjoint variables that allows maintaining a certain level in a reservoir by modifying the height of the dam weir, using one-dimensional and hydrological models for the flow. A sensitivity analysis is carried out to verify the capacity of the method implemented by means of simplified cases, in order to subsequently study a real domain in the Ebro river.

Keywords: CFD, shallow water, hydrodynamic, flood events, adjoint-based control method

AMS Classification: 76M12, 65M08, 37N10

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An efficient solver based on logically rectangular meshes for Biot's consolidation model

Javier Zaratiegui, Carmen Rodrigo[1], Andrés Arrarás, Laura Portero[2]

SUMMARY

The study of poroelasticity is of great interest in many societal relevant applications such as geothermal energy extaction, CO_2 storage or hydraulic fracturing, among others. In these models, there is a coupling between the fluid flow and solid deformation within a porous medium. Maurice Biot established in [1] a general three-dimensional mathematical formulation for these problems. When it comes to modeling real applications, numerical simulation becomes mandatory. Consequently, an intensive research has been carried out in the development of efficient discretizations as well as solution methods for the algebraic system that arises from Biot's poroelasticity model.

In this work, we consider a discretization of the quasi-static Biot's model based on the multipoint stress-multipoint flux mixed finite element method introduced in [2], which is locally conservative and can be formulated on simplicial and quadrilateral meshes. Moreover, it can handle accurately discontinuous full tensor permeabilities and Lamé coefficients, whose are the most common case related to subsurface flows.

The discrete scheme for Biot's model yields large systems of algebraic equations, so that its solution is a crucial aspect in numerical simulation, due to the fact that there is a huge computational cost involved in it. There are two main strategies employed to deal with the solution of such systems: monolithic methods and iterative coupling methods. In this talk, we will comment on these strategies and an efficient solver based on logically rectangular meshes for this problem will be proposed. This type of meshes improve the overall performance when structured data is used, since they take advantage of recent computer architectures. Finally, the robustness of the new solver is illustrated by presenting numerical results.

Keywords: poroelasticity, Biot's model, solver

AMS Classification: 65F10, 65N30, 65N20

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