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ANALYSIS OF BIFURCATIONS APPEARING IN THE NONLINEAR HELICOPTER FLIGHT DYNAMICS

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Abstract. The bifurcation theory is interested in the changes of the qualitative structure of dynamical system solutions when control parameters are varied. It is exploited here in order to analyse the highly nonlinear flight dynamics of a helicopter. This feature comes from the couplings between different constituting elements and physical variables and also from the overwhelming role of the main rotor whose dynamics is inherently quite complex. After describing the framework of the physical model i.e. the states, the control parameters and the dynamics, the appearing bifurcations are here analysed and characterised mathematically. Then the influence of the present nonlinearities over the global helicopter behaviour is assessed.

It is first shown that the formalism of a system of differential algebraic equations is here required so as to impose some algebraic constraints on some translational and rotational velocities, thus avoiding any inappropriate divergent movement. On the one hand, a bifurcation of equilibrium points associated to a real eigenvalue is linked to the vortex ring state phenomenon which occurs during steep descent flight. In this case, jumps and hysteresis reveal to be responsible for the dangerousness of such a situation. On the other hand, bifurcations of periodic orbits are observed and evaluated as triggering harmful pilot-aircraft couplings. Their type and characteristics are determined. To put in a nutshell, the nonlinear rotorcraft dynamics gives raise to interesting bifurcations whose description and characterisation need to be successfully performed in order to help avoiding dangerous configuration and recovering from these last ones.

Keywords: Bifurcation theory, dynamical systems, flight dynamics.

AMS classification: 34K18, 34K20.

Introduction

Helicopter flight dynamics is highly nonlinear because of its complex rotor dynamics and of the numerous physical couplings. In this paper, several types of bifurcations are studied and related to bifurcations of equilibrium points and of periodic orbits. The focus is stressed on the mathematical aspects of the analysis of a real rotorcraft behaviour.

First the mathematical framework must be defined and the type of mathematical equations involved must be made explicit. We can notice that the classical formulation of a system of ordinary differential equations does not fit well with this issue and that a system of differential algebraic equations must be employed. Secondly concrete bifurcations will be examined. On the one hand, a fold bifurcation of equilibrium points is diagnosed as underlying the vortex ring state phenomenon and gives raise to a hysteresis dynamics. On the other hand, bifurcations of periodic orbits trigger a jump in the oscillation amplitude and are responsible for harmful rotorcraft-pilot couplings. Finally it is shown how efficient the bifurcation theory can be for the analysis of nonlinear rotorcraft flight dynamics. This study presents the adaptations necessary in order to employ such a methodology in this particular case. It also gives concrete mathematical propositions and statements relative to helicopter flight dynamics.

§1. Mathematical modelling and numerical aspects for the helicopter flight dynamics analysis

Before being able to analyse such a dynamical system, it is first necessary to define the mathematical model i.e. to describe the dynamical system and the type of equations involved (and perhaps to complete also the modelling of the helicopter flight dynamics). Then numerical algorithms need to be developed and employed so as to calculate the characteristic loci of bifurcation theory.

1.1. Mathematical model

In order to define a dynamical system (1), we must specify the vector of state variables $X \in \mathbb{R}^n$, the vector of command variables (or control parameters) $U \in \mathbb{R}^k$ and the vector field corresponding to the state dynamics $F \in C^\infty\left(\mathbb{R}^n \times \mathbb{R}^k, \mathbb{R}^n\right)$ with $n, k \in \mathbb{N}^*$.

$$\dot{X} = F\left(X, U\right). \tag{1}$$

First the state variables describing the rotorcraft flight dynamics are:

$$X = \left(U_{hel}, V_{hel}, W_{hel}, P_{hel}, Q_{hel}, R_{hel}, \phi, \theta, Vim_{MR}, Vim_{TR}\right). \tag{2}$$

They correspond on the one hand to the classical variables of flight dynamics, i.e. U_{hel}, V_{hel}, W_{hel}, P_{hel}, Q_{hel}, R_{hel}, ϕ and θ, which are the translational velocities, the rotational velocities and the Euler angles. On the other hand, (Vim_{MR}, Vim_{TR}) are the (mean) induced velocities of the main and tail rotors which are specific rotorcraft variables. Secondly the helicopter has four controls:

$$U = \left(DT0, DTC, DTS, DTA\right). \tag{3}$$

The three first ones command the main rotor i.e. $DT0$ is the collective pitch, DTC the lateral cyclic pitch, DTS the longitudinal cyclic pitch, whereas the last one DTA is the collective pitch of the tail rotor. Thirdly the expression of dynamics function F results here from the fundamental principle of dynamics and from aerodynamics modelling works. It corresponds to

$$F\left(X, U\right) = \left(\dot{U}_{hel}, \dot{V}_{hel}, \dot{W}_{hel}, \dot{P}_{hel}, \dot{Q}_{hel}, \dot{R}_{hel}, \dot{\phi}, \dot{\theta}, \dot{V}im_{MR}, \dot{V}im_{TR}\right) \tag{4}$$

The physical model derives from Newton's laws of motion and is written in the body-fixed axes at the centre of gravity of the rotorcraft [8].

$$
\begin{aligned}
\dot{U}_{hel} &= -\left(W_{hel}\cdot Q_{hel} - V_{hel}\cdot R_{hel}\right) + \frac{F_X}{M_{hel}} - g\sin\theta,\\
\dot{V}_{hel} &= -\left(U_{hel}\cdot R_{hel} - W_{hel}\cdot P_{hel}\right) + \frac{F_Y}{M_{hel}} + g\cos\theta\sin\phi,\\
\dot{W}_{hel} &= -\left(V_{hel}\cdot P_{hel} - U_{hel}\cdot Q_{hel}\right) + \frac{F_Z}{M_{hel}} + g\cos\theta\cos\phi,\\
I_{XX}\dot{P}_{hel} &= \left(I_{YY} - I_{ZZ}\right)Q_{hel}\cdot R_{hel} + I_{XZ}\left(\dot{R}_{hel} + P_{hel}\cdot Q_{hel}\right) + M_X,\\
I_{YY}\dot{Q}_{hel} &= \left(I_{ZZ} - I_{XX}\right)R_{hel}\cdot P_{hel} + I_{XZ}\left(R_{hel}^2 - P_{hel}^2\right) + M_Y,\\
I_{ZZ}\dot{R}_{hel} &= \left(I_{XX} - I_{YY}\right)P_{hel}\cdot Q_{hel} + I_{XZ}\left(\dot{P}_{hel} - Q_{hel}\cdot R_{hel}\right) + M_Z.
\end{aligned}
\tag{5}
$$

The forces (F_X, F_Y, F_Z) and the moments (M_X, M_Y, M_Z) contains the contributions of the main rotor, the tail rotor, the fuselage, the horizontal tailplane, the vertical fin. These external forces are taken into account in addition to the weight (mass M_{hel}). $I_{XX}, I_{YY}, I_{ZZ}, I_{XZ}$ are the fuselage moments of inertia along the body reference axes.

The corner stone of the computation procedures linked to dynamical system problems consists in a so-called continuation algorithm. Such a software was developed for example by P. Guicheteau at ONERA for his studies on nonlinear fixed-wing aircraft flight dynamics [6, 7]. The continuation algorithm consists basically in the repetition of four steps: seeking a point on the solution curve, evaluating the tangent direction (Jacobian matrix calculation), predicting a new point and correcting the predicted point such that the calculated point is effectively on the curve. The characteristic loci can and must always be expressed under the form of an implicit system of n equations and $(n+1)$ variables (with $n \in \mathbb{N}^*$). As a consequence, there can only be one single control parameter.

In this study, the vortex ring state phenomenon will be examined. As a consequence, the focus is stressed on the dynamics along the vertical axis and the influence of a descent rate variation. The main rotor collective pitch $DT0$ which mainly governs V_Z and which determines the main rotor thrust is therefore selected as control parameter U. Unfortunately for a helicopter, all the physical variables are often coupled. When the collective pitch $DT0$ is reduced, the main rotor (torque) moment decreases also. But since the tail rotor still creates the same (anti-torque) moment as before, the helicopter begins to turn. To stabilise the yaw rate R_{hel} and to prevent the helicopter from turning, it is necessary to change the value of the tail rotor collective pitch DTA.

By imposing R_{hel} to zero by means of an additional algebraic constraint, the adapted trim value of the tail rotor collective pitch DTA is indirectly calculated. For equivalent reasons, the lateral velocity V_Y is forced to zero by determining the required lateral cyclic pitch angle DTC and the longitudinal cyclic pitch angle DTS is chosen such that the forward velocity V_X is equal to a fixed forward velocity V_{H0} (null here). Finally the movement can be imposed in a vertical plane by means of the following system of algebraic equations:

$$
\begin{cases}
R_{hel}\left(X, DT0, DTC, DTS, \mathbf{DTA}\right) = 0,\\
V_X\left(X, DT0, DTC, \mathbf{DTS}, DTA\right) = 0,\\
V_Y\left(X, DT0, \mathbf{DTC}, DTS, DTA\right) = 0.
\end{cases}
\tag{6}
$$

Thus, as a partial conclusion of this modelling part, for a helicopter flight dynamics problem, it can be stated that the description must be made by means of a system of **differential algebraic equations (DAE)**. The classical formulation under the form of a system of (autonomous) differential equations is not convenient.

1.2. Local bifurcations

The current study deals with local bifurcations of vector fields whose analysis is accomplished by examining the vector field in the neighbourhood of the (degenerate) equilibrium points or periodic orbits. The associated theory makes the assumption that considering the linearised system or truncated Taylor series of the vector fields allows to draw directly a conclusion for the nonlinear problem and the global (asymptotic) behaviour of its solutions. The methodology relies partly on the theorem 1.

Theorem 1 (Hartman-Grobman)**.** *If* $D_X F\left(\bar{X}\right)$ *has no zero or purely imaginary eigenvalues then there is a homeomorphism h defined on some neighbourhood* $\mathcal{U}$ *of* $\bar{X} \in \mathbb{R}^n$ *locally taking orbits of the nonlinear flow to those of the linear flow* $e^{tD_X F(\bar{X})}$*. The homeomorphism preserves the sense of orbits and can also be chosen to preserve parametrisation by time.*

Definition 1. If no eigenvalues of the Jacobian matrix $D_X F\left(\bar{X}\right)$ has a zero real part, then $\bar{X}$ is called a **hyperbolic** fixed point.

In such a situation, linearisation is sufficient to determine the asymptotic behaviour of solutions. On the contrary, when one of the eigenvalues has got a zero real part, the fixed point is said to be **nonhyperbolic**. Then it may be necessary to calculate higher order coefficients in the Taylor series and to evaluate the dynamics on the center manifold [5] so as to be able to conclude about the asymptotic behaviour of the overall system.

After describing the mathematical model and presenting elements concerning bifurcation theory, a concrete case is then examined with the help of this methodology.

§2. Fold bifurcation of equilibrium points, hysteresis (Vortex Ring State)

For a phenomenon such as the vortex ring state, the nonlinear behaviour comes from the nonlinear evolution of the induced velocity of the main rotor during descent flight. Indeed for a certain range of descent velocity (and forward speed), the rotor enters in its own wake and a doughnut-shaped ring appears around the rotor disk. The induced velocity of the main rotor increases strongly but its thrust is falling off and is not sufficient any more to stabilise the rotorcraft. The heave dynamics is affected, it corresponds approximatively to a change of sign of the derivative $Z_w = \partial \dot{W}_{hel} / \partial W_{hel}$ of the vertical dynamics and thus a behavioural change of the solution of the equation

$$\dot{W}_{hel} - Z_w W_{hel} = 0, \tag{7}$$

with the approximate analytic expression [8]

$$Z_W = -\frac{2 C_{Z_\alpha} \mathcal{A}_{blade}\, \rho\, (\Omega R)\, V_i / (\Omega R)}{\left(16 V_i / (\Omega R) + C_{Z_\alpha} N_{blade}\, c / (\pi R)\right) M_{hel}} \tag{8}$$

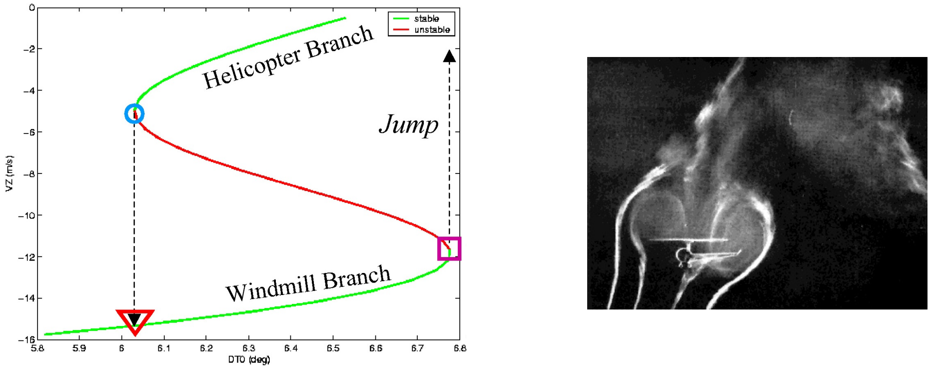

Figure 1: Bifurcation diagram associated to the vortex ring state [3] and presenting the descent rate V_Z in function of the collective pitch $DT0$

in hover or vertical flight and

$$Z_W = -\frac{C_{Z_\alpha} \mathcal{A}_{blade}\, \rho\, (\Omega R)}{2M_{hel}} \frac{4}{(8V_X/\,(\Omega R) + C_{Z_\alpha} N_{blade}\, c/\,(\pi R))} \tag{9}$$

in forward flight where C_{Z_α} is the blade lift curve slope, $\mathcal{A}_{blade}$ the blade surface, M_{hel} the total mass, N_{blade} the blade number, c the blade chord, Ω the nominal main rotor speed, R the main rotor radius and ρ the air density.

The bifurcation theory is interested in the determination of the bifurcation diagram (locus of equilibrium points), the locus of the bifurcation points and the equilibria surface. They are calculated in the following sections.

2.1. Locus of equilibrium points (bifurcation diagram)

The continuation algorithm allows to compute the bifurcation diagram of the system made of equations (1), (2), (3) and (6). Its result is shown in Figure 1.

According to [5], the "generic" saddle-node bifurcation looks qualitatively like the family of equations $\dot{x} = u - x^2$ in the zero eigenvector direction (and with hyperbolic behaviour in the complementary directions).

In Figure 1, the equilibrium curve contains two stable branches (green) and in the middle of them an unstable branch (red). The two bifurcations are linked to a zero real eigenvalue and are turning points [7]. For the range of control parameters $DT0$ between the two critical values, there are three equilibrium points whereas outside this region there is only one single equilibrium point. Such a bifurcation diagram is the typical one of a **hysteresis**. For flight dynamics engineers, the flight regimes at low descent rates is called "helicopter branch" and the one at high descent rates is named "windmill branch".

Concretely when the system is in a steady configuration near the bifurcation point, a little variation of the control parameter induces a situation where the system does not succeed any more in stabilising itself. As a consequence, a jump occurs on the other branch of equilibria

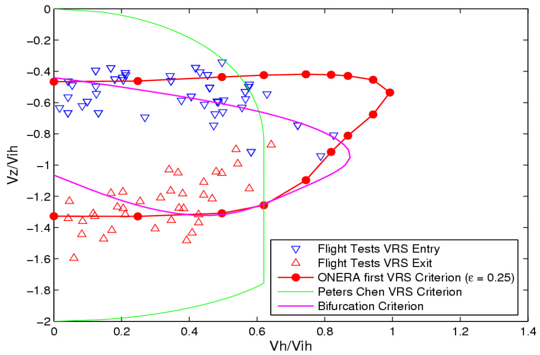

Figure 2: Boundaries of the vortex ring state region

(and isn't reversible with little opposite variations). From the viewpoint of flight dynamics, this jump from the helicopter branch to the windmill branch shows the loss of stability and the sudden increase in descent rate of the helicopter which appear when entering in vortex ring state.

After having determined the locus of equilibrium points and the type of dynamics associated to this bifurcation diagram, it is interesting to compute the locus of bifurcations points. It provides the analysis with new powerful information.

2.2. Locus of bifurcation points

The locus of bifurcation points is composed of the equilibria for which a behavioural change occurs (such as stability loss) that is to say here equilibria such that one real eigenvalue of the Jacobian matrix is equal to zero. The associated mathematical criterion is $\det\left(D_X\dot{X}\right) = 0$ which can also be written with the notation employed in the equation (1):

$$\det\left(D_X F\left(X, U\right)\right) = 0. \tag{10}$$

The continuation algorithm permits to compute the locus of bifurcation points by solving the following system of equations whose control parameters are V_{H0} and V_{Z0}:

$$\left\{\begin{array}{r}\dot{X} = F\left(X, U\right),\\ \det\left(D_X F\left(X, U\right)\right) = 0,\\ R_{hel}\left(X, DT0, DTC, DTS, \mathbf{DTA}\right) = 0,\\ V_X\left(X, DT0, DTC, \mathbf{DTS}, DTA\right) = V_{H0},\\ V_Y\left(X, DT0, \mathbf{DTC}, DTS, DTA\right) = 0,\\ V_Z\left(X, \mathbf{DT0}, DTC, DTS, DTA\right) = V_{Z0}.\end{array}\right. \tag{11}$$

In Figure 2, the locus of the bifurcation points (labelled "Bifurcation Criterion" and purple-coloured) is compared with data resulting from flight tests organised by ONERA at

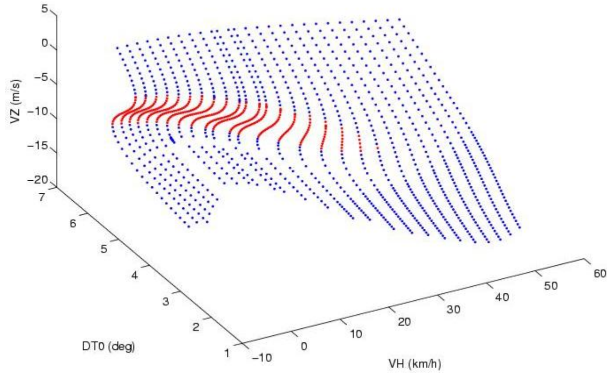

Figure 3: Surface of equilibria near the vortex ring state region

the French flight test centre of Istres and other criteria delimiting the VRS zone. The sudden drops are represented with blue triangles and the stabilisation points with red triangles.

As observed in the diagram presenting the forward velocity V_H and the descent rate V_Z normalised by the main rotor induced velocity in hover V_{ih} (cf. again Figure 2), the locus of bifurcation points fits well with the flight tests and predicts well the zone of instabilities.

Besides the gap for the lower frontier can be explained by the fact that the flight tests diagnose the conditions for which the aircraft stabilises after the drop whereas the bifurcation point represents the conditions for which the jump from the windmill branch to the helicopter branch occurs. The first point has got a bigger descent rate than the second one.

Moreover another relevant information can be obtained by scrutinising the surface of equilibrium points.

2.3. Surface of equilibria

Practically the surface of equilibria is actually determined by calculating the loci of equilibrium points for several longitudinal velocity V_H. The algebraic equations are (12) and the control parameters are $DT0$ and V_{H0}:

$$\begin{cases} R_{hel}\,(X, DT0, DTC, DTS, \mathbf{DTA}) = 0, \\ V_X\,(X, DT0, DTC, \mathbf{DTS}, DTA) = V_{H0}, \\ V_Y\,(X, DT0, \mathbf{DTC}, DTS, DTA) = 0, \end{cases} \tag{12}$$

The surface of equilibria in the neighbourhood of the vortex ring state is exposed in Figure 3. From the point of view of dynamical system theory, such a surface is called a **cusp** (cf. [5, page 355] or [9, pages 344-346]). There is a turning fold [7] i.e. a zone with three equilibrium points and another one with only one single equilibrium point. By considering the surface where the stable blue points are distinguished from the unstable red ones and by examining the configuration, an escape strategy can be deduced. When the aircraft jumps from the helicopter branch to the (windmill) branch with high descent rate, it is in a zone with three equilibriums. By increasing its forward velocity, the unstable zone reduces itself and disappears at the end. The helicopter is then in a zone with only one single stable equilibrium which means in a safe situation.

Conclusion about the analysis of a real bifurcation of equilibria in the case of the vortex ring state phenomenon

The mathematical formulation as a system of differential algebraic equations (DAE) seems to be necessary for the description and analysis of rotorcraft flight dynamics. Indeed many variables are coupled and some algebraic constraints must be added in order to avoid senseless configurations. As far as nonlinear analysis is concerned, on the one hand, the bifurcation theory reveals an underlying hysteresis phenomenon triggered by saddle-node bifurcations of equilibrium points. On the other hand, the locus of bifurcations points proves to be a relevant criterion in order to delimit the zone of instabilities linked to the vortex ring state [2].

This first part was devoted to the thorough analysis of a bifurcation of equilibria associated to a real eigenvalue and corresponding to the phenomenon of vortex ring state. After describing the mathematical model, the bifurcation diagram, the surface of equilibria and the locus of the bifurcation points were determined and interpreted from the both points of view of a mathematician and a flight dynamics engineer. In the next part, a bifurcation of periodic orbits will be studied. The mathematical model comes from the representation of a rotorcraft command channel with the use of the describing function theory.

§3. Bifurcation of limit cycles (Pilot-Induced Oscillations)

In order to perform the analysis of the rotorcraft command channel, the describing function method is employed and some elements about its mathematical justification is first introduced. Then the equations associated to the flight control system are made explicit. Finally the solution is computed thanks to the continuation algorithm and the results are interpreted with the bifurcation theory formalism.

3.1. Methodology

In order to exploit the describing function method [4], two conditions must hold. The first one states that there must be a clearly identifiable nonlinear element which can be isolated from the linear part whereas the second one stipulates that the linear part must behave like a low-pass filter. For the closed-loop system, the determination of the existence of possible periodic orbits and of their first-harmonic properties requires to solve the harmonic balance equation:

$$1 + L(j\omega) \cdot N(A, \omega) = 0, \tag{13}$$

where ω is the pulsation of the possible limit cycle, A the amplitude of its first harmonic, $N(A, \omega)$ the describing function of the nonlinear element (i.e. the rate-limited actuator) and $L(j\omega)$ the linear part including the bare airframe, the pilot and the linear actuators.

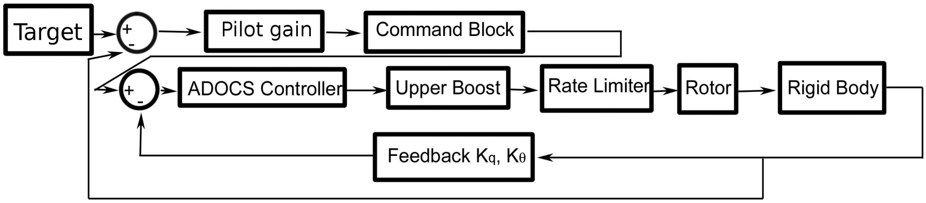

Figure 4: Closed-loop ADOCS command channel

Some elements relative to the mathematical foundation of the describing function method were explained in the previous section. The handled flight control system is exposed before beginning its concrete examination.

3.2. Command channel

The rotorcraft command channel presented in Figure 4 is the longitudinal one of the ADOCS helicopter as described by the NASA technical memorandum [10]. It contains the command block which filters the possible too aggressive pilot inputs, the blocks modelling the dynamics of the rotor and of the fuselage and the feedback loop block. The displacement velocity of the swashplates which command directly the motion of the main rotor blades is limited to 10 inches/s. This last one is here responsible for the observed nonlinear behaviour.

The longitudinal flight control system is analysed by means of the describing function method. According to (13), the equation (14) requires to be solved so as to diagnose the possible existence of a periodic orbit and to estimate the amplitude A and phase delay ϕ of its first harmonic for various values of input oscillation amplitude θ_c and for a pilot gain $K_p = 1$ (fixed nervousness here):

$$\Big(1 + Rotor \cdot RigidBody \cdot N(A, \omega) \cdot Actuator \cdot \big(K_p \cdot CommandBlock + Feedback\big)\Big) \times A \exp(j\phi) = Actuator \cdot CommandBlock \cdot \theta_c. \quad (14)$$

The characterisation of a saddle-node bifurcation of periodic orbits can be found in [9] and indeed it can be observed that Figure 5 is typical of a **saddle-node bifurcation of periodic orbits** [1, 9]. When the reference amplitude is increased from 0.33 rad to 0.34 rad, the amplitude of the entry state of the rate limiter jumps from 6 to 10.

Concretely the sudden increase may surprise, disturb greatly the pilot which does not succeed any more in controlling the aircraft, what leads to a risky situation.

Conclusion

During this study, two different phenomena coming from the field of rotorcraft flight dynamics were dealt with. Their underlying dynamics is governed by different types of bifurcations.

A fold bifurcation of equilibrium points of real eigenvalue proves to be responsible for a sudden jump of one branch of equilibria (helicopter branch) to another one (windmill branch)

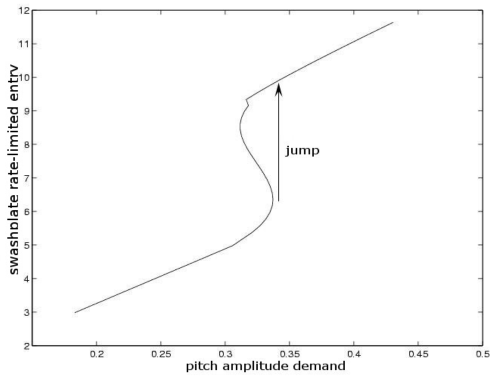

Figure 5: Jump of the oscillation amplitude

for a little variation of the (collective pitch) control. According to the bifurcation diagram, the underlying dynamics is a hysteresis. This last one explains mathematically the appearance of the vortex ring state phenomenon.

As far as pilot induced oscillations are concerned, a saddle-node bifurcation of periodic orbits is here observed. Amongst others, they imply jumps in amplitude of the periodic orbits and trigger some flying qualities cliffs.

Several important results were presented in this research paper. The detection of real bifurcations allows to delimit successfully the region of vortex ring state. The determination of the existence and of the properties of a bifurcation of limit cycles shows that the command channel of the ADOCS helicopter demonstrator which was adapted for this study is likely to have some flying qualities cliffs.

As a conclusion, the bifurcation theory reveals to be a useful tool for the nonlinear analysis of rotorcraft flight dynamics. It provides criteria helping delimiting dangerous regions of flight or detecting changes of flying qualities.

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