

Multiobjective Evolutionary Algorithms. Pareto Rankings

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Abstract

In this work we present two new Pareto based ranking methods. We compare them with three classical ones due to Belegundu, Goldberg and Fonseca and Fleming. Furthermore, we introduce the problem of classification errors. One of the proposed methods outperforms the others in five out of seven test problems.

Keywords: Multiobjective Evolutionary Algorithms. Pareto Ranking.

AMS Classification: 90C29, 90C59

1 Introduction

In this work, we study an important element in Multiobjective Evolutionary Algorithms (MOEAs): *the ranking of the population individuals*, in order to establish later the probabilities of survival that are necessary for the selection process.

We show how three classical ranking methods work, we present two new ones and we make a comparative study. At the end we introduce the problem of classification errors.

Evolutionary Algorithms (EAs) are heuristic methods for search and learning, based on the principles of natural evolution. A key aspect of EAs is that they evolve a population of potential solutions to a problem (individuals) instead of only one candidate. The population evolves by means of the selection, mutation and recombination processes of its individuals.

A common and very important element in EAs is the selection mechanism. This process determines which elements of the population are selected to be members of the next generation. It is done according to how good or bad a solution is. The better it is, the higher probability of survival it has, and so, it has a higher probability of being selected for the next generation. It is necessary to establish some criteria to determine if one solution is better than another. In the scalar case, it is obvious: the better objective value it has, the better a solution is. In the multiobjective case, there is not only one criterion to conclude whether one solution is better than another, because no total order exists in the set of solutions.

A classification of solving methods for Multiobjective Problems (MOP) can be done in function of the moment when the Decision Maker (DM) takes part in selecting the efficient solutions. The DM can take part before, during or after the optimization process. The researches developed in this paper are methods that are used, in general, in methods that use 'a posteriori' information, in which the aim of the process is to generate a set of efficient solutions. A reference about the different approaches can be found in [3] and [6].

The approach in which we are interested uses directly the concept of Pareto optimality to define the selection process. These methods rank the individuals in a manner such that the non dominated individuals of the population have a lower ranking, and so, a higher probability of being selected. These methods are known as Pareto ranking. Some of them can be found in [1], [2], [5] and [7].

2 Multiobjective Problems. Pareto Rankings

Let a Multiobjective Problem be

$$\begin{aligned} \min F(a) &= (f_1(a), \dots, f_q(a)) \\ \text{s.a:} \quad &a \in A \end{aligned}$$

where $A = \{a \in R^n | g_i(a) \leq 0, i = 1, \dots, m\}$, $f_j : R^n \rightarrow R, j = 1 \dots, q$ and $g_i : R^n \rightarrow R, i = 1, \dots, m$.

A is the feasibility region of the problem and its points are called feasible solutions to the problem. When these solutions are a part of a population, we will refer to them as individuals of the population.

In general, the objective functions are of a conflicting nature, improving an objective can make another objective worsen and this causes there to be no solution that simultaneously minimizes all the objectives. So, no ordering can be defined in a natural way (in function of the values of the objective functions) that let us say if one solution is better than another.

There are several approaches for solving MOP and all of them have a point in common, they look for solutions that are satisfactory for the DM from a set of solutions called efficient solutions.

Definition: Given two solutions $a^1, a^2 \in A$, we say that $a^1 = (a_1^1, \dots, a_n^1)$ dominates $a^2 = (a_1^2, \dots, a_n^2)$ if and only if $F(a^1)$ is partially less than $F(a^2)$, i.e., if and only if

$$f_i(a^1) \leq f_i(a^2) \forall i = 1, \dots, q \text{ and } \exists i \in \{1, \dots, q\} \text{ with } f_i(a^1) < f_i(a^2)$$

Definition: One solution $a \in A$ is said to be *Pareto optimal* or *efficient* with regard to a set $B \subset A$ if and only if there does not exist any $a' \in B$ such that a' dominates a .

It is clear from the second definition that the DM only looks for solutions to the problem that are non dominated, and among them, the DM selects those that are more

satisfactory.

Next, we describe briefly how three of the most commonly used methods work. One belongs to Belegundu [1], another to Goldberg [5] and the last one to Fonseca and Fleming [2].

- a) Belegundu’s ranking: All non dominated individuals are assigned rank 0 (or 1) and the dominated ones rank 1 (or 2).
- b) Goldberg’s ranking: It assigns equal probability of reproduction to all non dominated individuals in the population. The method consisted of assigning rank 1 to the non dominated individuals and removing them from contention, then finding a new set of non dominated individuals, ranked 2, and so forth.
- c) Fonseca and Fleming’s ranking: An individual’s rank corresponds to the number of individuals in the current population by which it is dominated. Non dominated individuals are, therefore, all assigned the same rank equal to zero, while the dominated ones have values between 1 and $k - 1$, where k is the population size.

Considering the example MOP2 introduced in [6], the landscapes of the ranking functions in a discretization of 1000 points in $-2 \leq x \leq 2$, $-2 \leq y \leq 2$ is shown in figure 1.

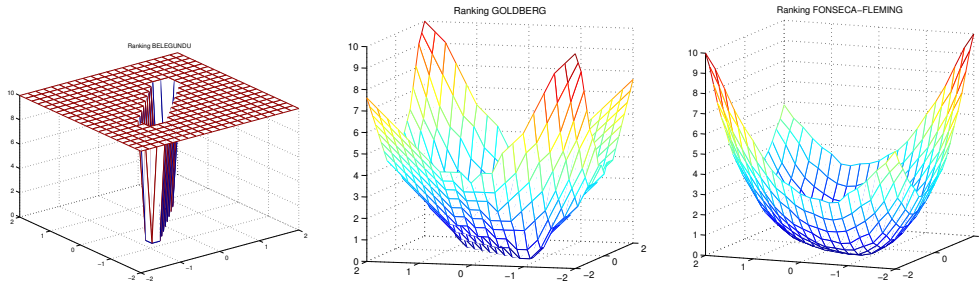


Figure 1

3 New Pareto Rankings

In this section we present two new methods for ranking solutions. Both have in common the same ideas as the previous ones. They establish a ranking that gives the lowest values to the efficient solutions and they increase the ranking when the solutions are further, in any sense, from the efficient ones.

AM1 Method

This method transforms the solution vectors $(f_1(a), \dots, f_q(a))$ in scalar values by means of an aggregation with random weights $\mathcal{U}(0, 1)$.

Given the current population, P , q random weights are generated. The value of the aggregated function is obtained for each one of the individuals. Then, for each individual, it is determined how many individuals have a better value of the aggregated function (in a similar way to Fonseca-Fleming’s ranking determines, for each individual, how many

individuals dominate him). This process is repeated a prefixed number of times. Basically, the method determines as best solutions those that are preferred under more points of view (best for a bigger set of random weights) To guarantee that the efficient individuals are given rank 1, they are identified in a first step of the algorithm and removed from the population before the process starts. Then, the process is applied on the non efficient individuals.

The schema of the algorithm is shown in figure 2 a), in which it is assumed that the efficient individuals have been removed from the population.

The function $rand()$ returns $\mathcal{U}(0, 1)$ values and the function $sort()$ returns a vector in which the h -th component contains the index of the vector \bar{F} that occupies the h -th position after sorting it from the lowest to the highest value. As we have pointed out before, M is the number of times that the process of selecting random weights is repeated.

AM2 Method

This method uses Goldberg and Fonseca and Fleming's ideas together, and is a refinement of Goldberg's method. Initially, Goldberg's ranking is used to separate the individuals into layers. Then, the individuals of each layer are ranked using the number of individuals in the preceding layer that dominate them. So, in each layer the individuals are ranked with the information contained in the preceding layer.

Let C_h , $h = 1, \dots, H$ the layers defined by Goldberg's ranking such that $R_G(a_i) = h \forall a_i \in C_h$ and let $a_i^h \in C_h$, where $i = 1, \dots, |C_h|$.

The schema of the algorithm is shown in figure 2 b).

<pre> for i = 1 to k R_{AM1}(a_i)=0 end for for j = 1 to M U = (U₁, U₂, ..., U_q)' = = (rand(), ..., rand())' for i = 1 to k $\bar{F}(a_i) = F(a_i) \times U$ end for index=sort(\bar{F}) for i = 1 to k l =index(i) R_{AM1}(a_l)=R_{AM1}(a_l)+i - 1 end for end for </pre>	<pre> for i = 1 to C₁ R_{AM2}(a_i¹) = 1 end for acum = 1 for h = 2 to H for i = 1 to C_h R_{AM2}(a_i^h) = acum for j = 1 to C_{h-1} if a_j^{h-1} dominates a_i^h R_{AM2}(a_i^h) = R_{AM2}(a_i^h) + 1 end if end for end for acum = max_{i=1,..., C_h} {R_{AM2}(a_i^h)} end for </pre>
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Figure 2 a). AM1 method

Figure 2 b). AM2 method

In figure 3, the landscapes of the normalized ranking function are presented for the test problem MOP2 for both methods (in the AM1 method, we have considered $M=20$)

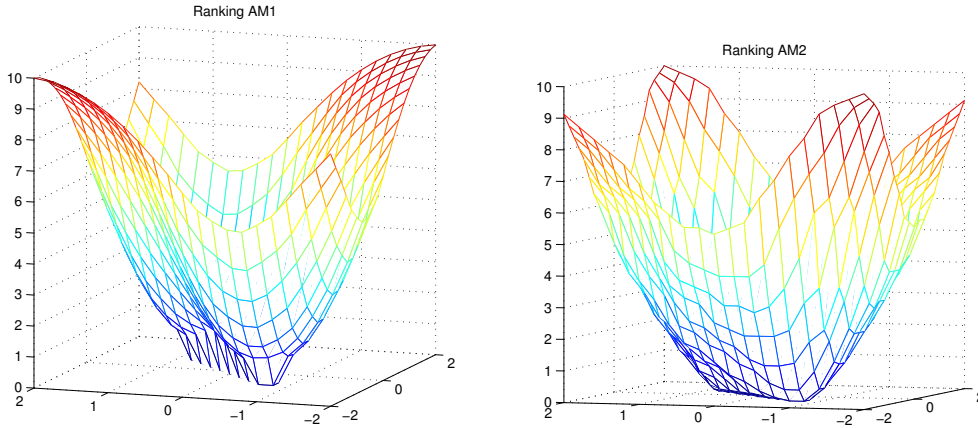


Figure 3

4 Comparison of Methods. Classification errors

With regard to the relationships among the rankings generated, we can establish the following characteristics:

- a) In all of them, the existing and the new ones, the efficient solutions get the lowest ranking, and starting from there, the ranking of the rest of the solutions is established in an increasing way. The further a solution is from the efficient ones (in any sense), the higher ranking this solution gets.
- b) The discrimination capacity of these methods for a population P with $|P| = k$, from minor to major is:

- *Belegundu's ranking.* $\forall a_i \in P \subset A, R_B(a_i) \in \{0, 1\}$.
- *Goldberg's ranking, Fonseca-Fleming's ranking and AM2.* $\forall a_i \in P \subset A, R_G(a_i), R_{FF}(a_i), R_{AM2}(a_i) \in \{0, \dots, k\}$.
- *AM1 ranking.* $\forall a_i \in P \subset A, R_{AM1}(a_i) \in \{0, \dots, M \times k\}$.

Then, the AM2 method equals (or improves, for Belegundu's ranking) the discrimination capacity of the existing ones, and the AM1 method, can provide the smoothest ranking. We also have to take into account that Fonseca-Fleming's ranking, in general, and in spite of the fact that it has the same rank of variation as Goldberg's ranking, generates rankings with a higher capacity of discrimination ([4]). With respect to the AM2 method, it is a refinement of Goldberg's ranking, and so it will generally provide a higher capacity of discrimination.

c) In general, no relation among the rankings exists, except for the obvious one:

- Given $a_i, a_j \in P \subset A$ such that $R_B(a_i) < R_B(a_j)$ then $R_*(a_i) < R_*(a_j)$ where $*$ represents any of the other rankings.

If we compare the graphics presented before, we can observe that there is no compatibility between Goldberg's and Fonseca-Fleming's rankings. Given a set of alternatives, we can assign different ranking relations among them, that is to say, there can exist $a_i, a_j \in P \subset A$ such that $R_G(a_i) < R_G(a_j)$ and $R_{FF}(a_i) > R_{FF}(a_j)$ and viceversa.

With respect to the rankings we present, the situation is similar, except for Goldberg's ranking and AM2. The following result can be established immediately:

- Given $a_i, a_j \in P \subset A$ such that $R_G(a_i) < R_G(a_j)$ then $R_{AM2}(a_i) < R_{AM2}(a_j)$

Another important point corresponds to the errors produced when ranking a sample (a population for the evolutionary algorithm) instead of all the individuals of the feasibility region. As we show in figure 4, all methods can present, in some situations, ranking errors with respect to the sorting when the information of the global feasibility region is considered. In this figure we can observe that there is no monotone behaviour of the sample's ranking with respect to the order obtained when the global feasibility region is considered.

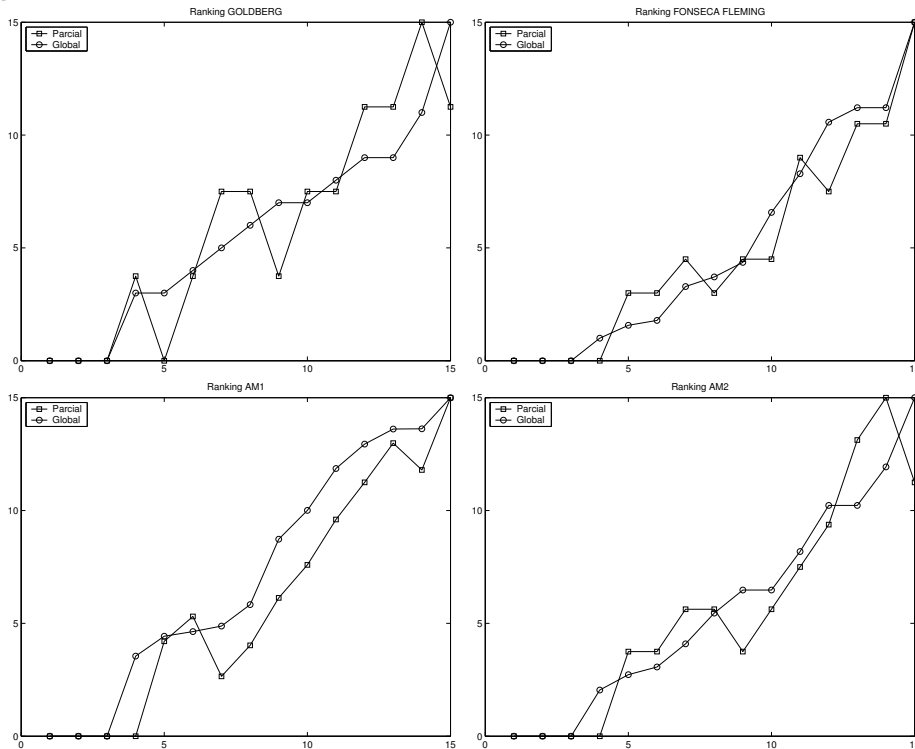


Figure 4.

Although this population is classified *relatively well* by all the methods presented, there can exist cases in which this might not happen, as we show in figure 5.

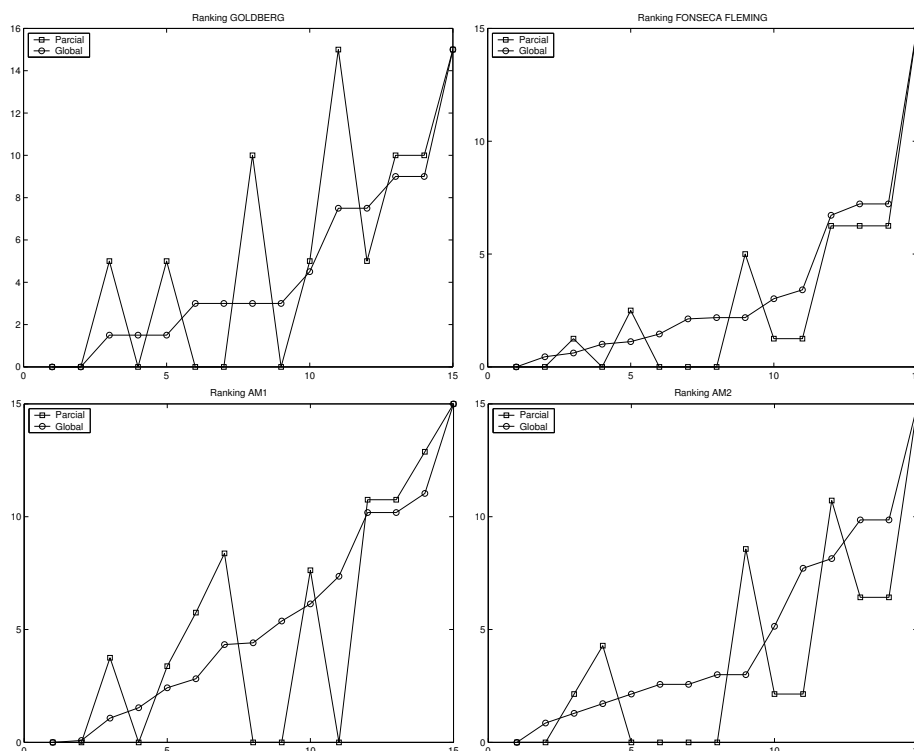


Figure 5.

To examine the better or worse behaviour of the former rankings, we have taken the seven test problems, MOP1, . . . , MOP7, proposed in [6] and we have generated, for each problem, 4 sets of 500 populations with sizes 10, 20, 30 and 40. For each of these 14000 populations, we have determined its ranking with every method and the ranking when considering the population in the global feasibility region (discretization of around 1000 points). After this, to measure the ranking error, Spearman's correlation coefficient has been calculated between both rankings and for every population size. Next, we present the mean values and the sum of the squares of the deviations to the mean of this coefficient in figure 6.

	Goldberg	Fons-Fleming	AM1	AM2		Goldberg	Fons-Fleming	AM1	AM2
MOP1 (10) \bar{x}	0,936	0,952	0,972	0,941	MOP5 (10) \bar{x}	0,648	0,706	0,646	0,659
ns^2	1,811	1,114	0,408	1,534	ns^2	37,50	24,67	67,41	39,31
(20) \bar{x}	0,965	0,980	0,989	0,966	(20) \bar{x}	0,741	0,817	0,831	0,774
ns^2	0,401	0,117	0,029	0,367	ns^2	20,19	11,09	31,92	17,39
(30) \bar{x}	0,971	0,985	0,993	0,971	(30) \bar{x}	0,818	0,883	0,914	0,849
ns^2	0,384	0,080	0,008	0,372	ns^2	7,991	3,401	8,143	5,683
(40) \bar{x}	0,976	0,990	0,995	0,976	(40) \bar{x}	0,842	0,910	0,942	0,869
ns^2	0,248	0,0328	0,004	0,241	ns^2	6,850	2,336	5,112	4,949

Figure 6.

	Goldberg	Fons-Fleming	AM1	AM2		Goldberg	Fons-Fleming	AM1	AM2
MOP2 (10) \bar{x}	0,890	0,931	0,973	0,888	MOP6 (10) \bar{x}	0,816	0,904	0,861	0,820
ns^2	4,204	1,941	1,502	3,810	ns^2	13,02	4,244	12,86	12,11
(20) \bar{x}	0,922	0,964	0,992	0,923	(20) \bar{x}	0,850	0,951	0,932	0,851
ns^2	0,964	0,331	0,0528	0,861	ns^2	4,810	0,519	2,833	4,617
(30) \bar{x}	0,936	0,976	0,995	0,937	(30) \bar{x}	0,863	0,965	0,951	0,864
ns^2	0,624	0,113	0,011	0,560	ns^2	3,593	0,212	1,726	3,396
(40) \bar{x}	0,946	0,982	0,997	0,948	(40) \bar{x}	0,875	0,974	0,966	0,876
ns^2	0,421	0,050	0,003	0,374	ns^2	2,399	0,119	0,802	2,312
MOP3 (10) \bar{x}	0,734	0,822	0,813	0,746	MOP7 (10) \bar{x}	0,855	0,913	0,915	0,866
ns^2	20,39	65,72	13,74	22,07	ns^2	6,544	2,664	4,969	5,655
(20) \bar{x}	0,798	0,887	0,909	0,806	(20) \bar{x}	0,893	0,949	0,963	0,900
ns^2	10,28	2,586	2,438	9,380	ns^2	0,893	0,949	0,963	0,900
(30) \bar{x}	0,835	0,920	0,937	0,843	(30) \bar{x}	0,914	0,964	0,977	0,920
ns^2	7,374	1,052	1,138	6,465	ns^2	1,304	0,494	0,260	1,084
(40) \bar{x}	0,861	0,941	0,953	0,868	(40) \bar{x}	0,927	0,975	0,984	0,932
ns^2	5,525	0,555	0,566	4,950	ns^2	0,576	0,129	0,106	0,527
MOP4 (10) \bar{x}	0,630	0,664	0,550	0,641					
ns^2	17,77	27,20	27,76	15,93					
(20) \bar{x}	0,714	0,752	0,672	0,726					
ns^2	9,191	6,985	13,24	8,458					
(30) \bar{x}	0,762	0,814	0,741	0,776					
ns^2	5,109	3,440	7,230	4,650					
(40) \bar{x}	0,789	0,840	0,778	0,804					
ns^2	3,969	2,277	5,237	3,507					

Figure 6. Cont.

As we gather from this figure, Goldberg’s method is the one that, in general, produces higher classification errors. In almost every case, the AM2 method slightly improves Goldberg’s method, as it was expected because AM2 is a refinement of Goldberg’s.

The AM1 method and Fonseca-Fleming’s method share the best behaviour. The AM1 method is the best in 5 of the 7 test problems. Furthermore, AM1 and Fonseca-Fleming’s methods are those which have a smaller variance and this means that these methods are more stable in the classification.

5 Conclusions

In this work, we have tackled an important element in the design of Multiobjective Evolutionary Algorithms: the construction of rankings for the individuals of the population to later establish the probabilities of survival.

We have taken two of the most usual ranking methods, belonging to Goldberg and Fonseca and Fleming; and a third one for its simplicity, belonging to Belegundu. These methods have been presented together with two new ones. All these methods have in common that the efficient solutions have the best (lowest) ranking. For the other solutions, the further they are from the efficient set, the higher ranking they get.

The AM1 method gives a higher discrimination capacity than the existing ones, because the rank of values is a positive multiple (integer) of the population size. The other, AM2, equals the discrimination capacity of Goldberg’s and Fonseca-Fleming’s methods.

On the whole, there is no compatibility among the rankings, except for Belegundu's ranking with respect to all the other methods, and Goldberg's ranking with respect to the AM2 method.

Finally, in 5 out of the 7 test problems, the proposed AM1 method outperforms the others, and the AM2 method only outperforms Goldberg's method.

Lastly, with the test done on ranking errors, we guess the necessity of future research to be done in a theoretical way on one of the topics in this paper: the classification errors and the designing of ranking methods which, alone or interacting with other elements of evolutionary algorithms, guarantee ranking errors as small as possible or even their diminution in successive iterations of the algorithms.

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