Rate of numerical difusion of finite volume schemes

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SUMMARY

Dissipation rates of Godunov upwind type schemes are often studied either through Fourier analysis or through the so-called modified equation technique. However, these techniques are limited to uniform cartesian meshes. In this work, we introduce a method based on energy estimates that allows to derive explicit lower bounds for the dissipation rate of semi-discrete Godunov type schemes based on non uniform cartesian, or on triangular meshes. We illustrate our purpose by applying the theory to the one-dimensional transport equation discretized by the upwind scheme on non-uniform meshes and to the two-dimensional wave equation discretized by the Godunov scheme on triangular meshes. On rectangular meshes, we also study a variant of the Godunov scheme which preserves a discrete variant of the wave invariant space. Several steps are used to obtain these results. First, the discrete solutions of the schemes are decomposed as sums of two elements: the first in the kernel of the discrete transport and wave operators and the other in the orthogonal of these kernels. Then it is verified whether these orthogonal subspaces are stable by the discrete operators. Finally, discrete Poincaré inequalities in the orthogonal subspaces are used to obtain the dissipation lower bounds. The best provable lower bounds are in addition shown to be solutions of an eigenvalue problem.

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