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Semigroup theory for the Stokes operator with Navier boundary condition on L^p spaces

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SUMMARY

We consider the motion of a viscous incompressible fluid given by non-stationary Navier-Stokes equation with slip boundary condition in a bounded domain

$$\begin{cases} \dfrac{\partial \boldsymbol{u}}{\partial t} - \Delta \boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} + \nabla\pi = \boldsymbol{0}, \quad \text{div } \boldsymbol{u} = 0 & \text{in } \Omega\times(0,T); \\ \boldsymbol{u}\cdot\boldsymbol{n} = 0, \quad 2[(\mathbb{D}\boldsymbol{u})\boldsymbol{n}]_{\boldsymbol{\tau}} + \alpha\boldsymbol{u}_{\boldsymbol{\tau}} = \boldsymbol{0} & \text{on } \Gamma\times(0,T); \\ \boldsymbol{u}(0) = \boldsymbol{u}_0 & \text{in } \Omega. \end{cases} \tag{1}$$

Here Ω is a bounded domain in $\mathbb{R}^3$ with boundary Γ. The initial velocity $\boldsymbol{u}_0$ and the (scalar) friction coefficient α are given functions; The external unit normal vector on Γ is denoted by $\boldsymbol{n}$, $\mathbb{D}\boldsymbol{u} = \frac{1}{2}\left(\nabla\boldsymbol{u} + \nabla^T\boldsymbol{u}\right)$ denotes the strain tensor and the subscript $\boldsymbol{\tau}$ denotes the tangential component **i.e.** $\boldsymbol{v}_{\boldsymbol{\tau}} = \boldsymbol{v} - (\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{n}$ for any vector field $\boldsymbol{v}$. The functions $\boldsymbol{u}$ and π describe respectively the velocity and the pressure of the fluid.

The boundary condition in (1) was introduced by H. Navier (in [1]) which is in recent years widely studied because of its significance in real world in different model for simulation of flows and fluid-solid interaction problems (cf. [2]).

The well-posedness of the above system imposing minimal regularity on α will be discussed. We use semigroup theory to first study the weak and strong solutions for the associated Stokes operator. Resolvent estimate uniform with respect to α is deduced which enables us to have bounds on the solution $\boldsymbol{u}$ of (1) independent of α. Finally we study the behaviour of the solution of (1) with respect to the friction coefficient, in particular what happens if α goes to ∞.

Keywords: Navier-Stokes equation, slip boundary condition, semigroup theory, dependence on friction coefficient

References

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