

Asymptotic behaviour of the Swallowtail catastrophe

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SUMMARY

We analyze the integral representation of swallowtail catastrophe $\Psi(x, y, z) := \int_{-\infty}^{\infty} e^{i(t^5 + xt^3 + yt^2 + zt)} dt$ for large values of one of the variables and bounded values of the other two. The integrand of this integral oscillates wildly and the asymptotic analysis is subtle. The standard saddle point method is complicated, therefore we use the simplified saddle point method introduced in (cf. [1]). The analysis is more straightforward with this method and it is possible to derive complete asymptotic expansions of $\Psi(x, y, z)$ for large values of the asymptotic variable and fixed values of the rest. The asymptotic analysis requires the study of different regions separated by the corresponding Stokes lines. The accuracy and the asymptotic character of the approximations is illustrated with some numerical experiments.

Keywords: Swallowtail integral, Asymptotic expansions, Modified saddle point method.

AMS Classification: 33E20, 41A60

References

- [1] J. L. López, E. Pérez Sinusía and P. Pagola. A systematization of the saddle point method. Application to the Airy and Hankel functions, *J. Math. Anal. Appl.* **354**, 347–359, 2009.

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