Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
Jaca, September 10–12th 2018

Hölder, Sobolev, weak-type estimates in mixed-norm with weights for parabolic equations

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SUMMARY

We prove weighted mixed-norm $L_t^q(W_x^{2,p})$ and $L_t^q(C_x^{2,\alpha})$ estimates for $1 < p, q < \infty$ and $0 < \alpha < 1$ and weighted mixed weak-type estimates for $q = 1$, as well as a.e. pointwise formulas for derivatives, for solutions $u = u(t, x)$ to parabolic equations of the form

$$\partial_t u - a^{ij}(t)\partial_{ij}u + u = f \quad t \in \mathbb{R},\ x \in \mathbb{R}^n$$

and for the Cauchy problem

$$\begin{aligned} \partial_t v - a^{ij}(t)\partial_{ij}v + v &= f \qquad \text{for } t > 0,\ x \in \mathbb{R}^n \\ v(0, x) &= g \qquad \text{for } x \in \mathbb{R}^n. \end{aligned}$$

The coefficients $a(t) = (a^{ij}(t))$ are just bounded, measurable, symmetric and uniformly elliptic. Furthermore, we show strong, weak type and BMO estimates with parabolic Muckenhoupt weights. It is quite remarkable that most of our results are new even for the classical heat equation

$$\partial_t u - \Delta u + u = f.$$

Keywords: Heat equation, weighted Sobolev estimate, mixed-norm estimate, semigroups.

AMS Classification: Primary: 35K10, 35B45, 42B37; Secondary: 58J35, 42B20

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