Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
Jaca, September 10–12th 2018

On defect of compactness for Sobolev embeddings

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SUMMARY

Defect of compactness for an continuous embedding of a Banach space X into a metric space Y is the difference between a weakly convergent sequence $u_k \in X$ and its weak limit u, taken modulo sequences vanishing in Y . For many pairs of spaces one knows a group G of isometries on X that allows to represent defect of compactness as profile decomposition - a sum of mutually decoupled terms of the form $g_k w$ with $g_k \in G$. Element $w \in X$ is given as the weak limit of $g_k^{-1} u_k$ and is called a concentration profile. Existence of profile decomposition in presence of a group has been proved for general Banach spaces. Profile decompositions are applied in PDE to prove convergence of putative approximate solutions when one has no compact embedding at hand. In such settings concentration profiles satisfy some equation at infinity and there may be reasons, e.g. Liouville theorems, for them to be zero, resulting in zero defect of compactness and thus convergence. In this talk we outline several recent results concerning Sobolev spaces on manifolds, where profile decomposition is constructed without a group of isometries and some applications to semilinear elliptic problems on Riemannian manifolds. The main part of this work is done jointly with Leszek Skrzypczak.

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