

A higher-order method on a graded mesh for a time-fractional diffusion problem

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SUMMARY

Alikhanov's high-order scheme for Caputo fractional derivatives of order $\alpha \in (0, 1)$ is generalised to nonuniform meshes and analysed for initial-value problems (IVPs) and initial-boundary value problems (IBVPs) whose solutions display a typical weak singularity at the initial time. It is shown that, when the mesh is chosen suitably, the scheme attains order $3 - \alpha$ convergence for the 1-dimensional IVP and second-order convergence for the IBVP. For the IBVP we consider the case where the spatial domain is the unit square and use a spectral method to discretise in space, but other spatial domains and other spatial dimensions and discretisations are possible. Numerical results demonstrate the sharpness of the theoretical convergence estimates.

The talk is aimed at non-specialists but it does assume some familiarity with finite difference methods.

Keywords: Caputo fractional derivative, Alikhanov scheme

AMS Classification: 65M06, 65M70

References

- [1] HU CHEN & MARTIN STYNES. Error analysis of a second-order method on fitted meshes for a time-fractional diffusion problem. <https://www.researchgate.net/publication/325245680>

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