Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
Jaca, September 10–12th 2018

Uniformly convergent expansions of the Struve functions in terms of elementary functions

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SUMMARY

Struve functions are solutions to the non-homogeneous Bessel's differential equation

$$\frac{d^2w}{dz^2} + \frac{1}{z}\frac{dw}{dz} + \left(1 - \frac{\nu^2}{z^2}\right)w = \frac{(z/2)^{\nu-1}}{\sqrt{\pi}\Gamma(\nu + 1/2)}.$$

They are useful in the description of several phenomena in aerodynamics, quantum mechanics, optical diffraction and other physical areas.

The approximation of these functions in terms of elementary functions is quite convenient in the analysis of those physical phenomena. The most commonly used approximations are the Taylor and the asymptotic expansions, useful for small and large values of $|z|$ respectively. But these approximations are not uniformly valid for $\Re z \in \mathbb{R}$. Then, we derive a convergent series expansion of the Struve functions in terms of elementary functions of z that, conveniently scaled, hold uniformly in $\Re z \in \mathbb{R}$.

The starting point of the analysis is a convenient integral representation. Then, the Taylor expansion of an appropriate factor of the integrand is used. The uniform expansions derived are accompanied by realistic error bounds and are compared with the Taylor and asymptotic expansions.

Keywords: Struve functions; convergent expansions; uniform expansions.

AMS Classification: 33C47; 41A58

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