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An efficient uniformly convergent method for solving singularly perturbed semilinear reaction-diffusion systems

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SUMMARY

In this talk we develop and analyze a numerical method for solving 1D semilinear parabolic singularly perturbed systems of reaction-diffusion type, which are given by

$$\begin{cases} \dfrac{\partial \mathbf{u}}{\partial t}(x,t) - \mathcal{D}_\varepsilon \dfrac{\partial^2 \mathbf{u}}{\partial x^2}(x,t) + \mathcal{A}(x,t,\mathbf{u}) = \mathbf{0}, \ (x,t) \in (0,1) \times (0,T], \\ \mathbf{u}(0,t) = \mathbf{g_1}(t), \ \mathbf{u}(1,t) = \mathbf{g_2}(t), \ \forall\, t \in (0,T], \ \mathbf{u}(x,0) = \varphi(x), \ \forall\, x \in [0,1], \end{cases}$$

being $\mathbf{u} = (u_1, u_2, \ldots, u_n)^T$, $\mathcal{D}_\varepsilon =$ diag $(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)$ satisfying $0 < \varepsilon_1 \le \varepsilon_2 \le \ldots \le \varepsilon_n \le 1$ and the nonlinear reaction term $\mathcal{A}(x,t,\mathbf{u}) = (a_1(x,t,\mathbf{u}), a_2(x,t,\mathbf{u}), \ldots, a_n(x,t,\mathbf{u}))^T$ is composed by sufficiently smooth functions a_i such that, for all $(x,t.\mathbf{v}) \in [0,1] \times [0,T] \times \mathcal{R}^n$, it holds that

$$\begin{aligned} &\frac{\partial a_i}{\partial v_j}(x,t,\mathbf{v}) \le 0, \ i \ne j, \ i,j = 1,\ldots,n, \\ &\sum_{j=1}^{n} \frac{\partial a_i}{\partial v_j}(x,t,\mathbf{v}) \ge \alpha > 0, \ i = 1,\ldots,n. \end{aligned}$$

In the case of having small diffusion parameters ε_i with different orders of magnitude overlapping boundary layers appear close to the end points of the interval $(0,1)$.

The numerical method which we propose combines a linearized version of the fractional implicit Euler method together with a splitting by components, to discretize in time, and the central finite difference scheme on an appropriate piecewise uniform mesh, to discretize in space. In this way, only small tridiagonal linear systems are involved in the numerical integration in time.

It is proven and tested that the proposed numerical algorithm is uniformly convergent, of first order in time and of almost second order in space.

Keywords: semilinear parabolic systems, reaction-diffusion, splitting, Shishkin meshes, uniform convergence.

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