On a bounded distribution derived from the shifted Gompertz law

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SUMMARY

A two-parameter distribution with bounded domain is obtained by means of an exponential transformation of the shifted Gompertz model (cf. [1]). The new distribution has probability density function given by

$$f(x; \alpha, \beta) = \beta \left[1 + \alpha(1 - x^\beta) \right] x^{\beta-1} \exp \left(-\alpha x^\beta \right), \qquad 0 < x < 1, \tag{1}$$

where $\alpha > 0$ and $\beta > 0$ are two shape parameters.

The main statistical properties of the distribution under consideration are provided in closed form. The moments are written in terms of the incomplete gamma function and the quantile function in terms of the Lambert W function. The order statistics are also studied and, in particular, the new model belongs to the do main of maximal/minimal attraction of the Weibull law. Moreover, the members of the new family of distributions can be ordered in terms of the hazard rate order.

The parameter estimation is carried out by the methods of maximum likelihood , least squares, weighted least squares and quantile least squares. The performa nce of these methods is assessed by means of a Monte Carlo simulation study. A symptotic confidence intervals based on maximum likelihood together with par ametric bootstrap confidence intervals are discussed and the coverage probabilit y and average width of these intervals are studied.

The random variable defined by (1) is useful to model real data taking values in the standard unit interval, such as percentages and p roportions, and its practical usefulness is illustrated by means of several real data sets.

The proposed distribution may provide a better fit than other two-parameter distributions, such as the classical beta and Kumaraswamy distributions as well as other models recently introduced (cf. [2] and [3]).

Keywords: Beta, Kumaraswamy, shifted Gompertz, bounded support.

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References

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