

## Raviart-Thomas finite elements of Petrov-Galerkin type

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### SUMMARY

The mixed finite element method for the Poisson problem with the Raviart-Thomas elements of low-level can be interpreted as a finite volume method with a non local gradient. In this contribution, we propose a variant of Petrov-Galerkin type for this problem to ensure a local computation of the gradient at the interfaces of the elements. The shape functions are the Raviart-Thomas finite elements. Our goal is to define test functions that are in duality with these shape functions: Precisely, the shape and test functions will be asked to satisfy a  $L^2$ -orthogonality property. The general theory of Babuška brings necessary and sufficient stability conditions for a Petrov-Galerkin mixed problem to be convergent. We propose specific constraints for the dual test functions in order to ensure stability. With this choice, we prove that the mixed Petrov-Galerkin scheme is identical to the four point finite volumes scheme of Herbin, and to the mass lumping approach developed by Baranger, Maitre and Oudin. Finally, we construct a family of dual test functions that satisfy the stability conditions. Convergence is proven with the usual techniques of mixed finite elements.

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