Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
Jaca, September 10–12th 2018

Abstract fractional differential equations with order varying in time in complex Banach spaces and its time discretization: Well–posedness, regularity, and asymptotic behavior

E. Cuesta[1], **R. Ponce**[2],

SUMMARY

An abstract time fractional differential equation with order varying in time

$$u(t) = u_0 + \partial_t^{-\beta(t)} Au(t) + f(t), \qquad t > 0, \tag{1}$$

is considered, where $u_0 \in X$, X is complex Banach space, $\beta : (0, +\infty) \to (1, 2)$ is the integration order, the linear operator $\partial_t^{-\beta(\cdot)}$ stands fractional integral in time of order $\beta(\cdot)$, and $A : D(A) \subset X \to X$ is an unbounded linear operator of sectorial type. For the sake of the simplicity it is assumed that $f \equiv 0$.

First of all a discussion on the convenience of a definition of the operator $\partial_t^{-\beta(\cdot)}$ versus some others considered in the literature is shown.

Once a convenient definition is chosen, conditions on $\beta(\cdot)$ for the well–posedness of (1) are stated in the framework of sectorial operators [1]. Under these requirements the asymptotic behavior and the regularity of the solution u is studied. Moreover, the asymptotic behavior of a convolution quadrature based time discretization is studied as well.

All these results extend the corresponding results for the constant order case [2], i.e. the case of $\beta(t) \equiv \beta \in (1, 2)$ is constant.

Theoretical results are numerically illustrated by means of several practical experiments.

Keywords: Fractional integrals; variable order; asymptotic behavior.

AMS Classification: 45A05, 45E10, 45N05.

References

[1] A. Pazy. *Semigroups of Linear Operators and Applications to Partial Differential Equations*. Springer-Verlag, New York, 1983.

[2] E.Cuesta, and R. Ponce. Well posedness, regularity, and asymptotic behavior of the continuous and discrete solutions of linear fractional integro–differential equations with order varying in time. (Submitted 2018).

[1]Department of Applied Mathematics
University of Valladolid (Spain)
email: eduardo@mat.uva.es

[2]Institute of Mathematics and Physics
University of Talca (Chile)
email: rponce@inst-mat.utalca.cl