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The piston problem: Modelling, Analysis and Control

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SUMMARY

We consider various models for the motion of a piston in a cylinder filled with a viscous gas. We begin by a toy model, in which the gas is modelled by the viscous Burgers equations. in this case we recall the global well-posedness and controllability results from [1] and [6]. We next discuss the case in which the fluid is modelled by the viscous Navier-Stokes equations, see [2].

The main part of the talk is devoted to new results (with E. Feireisl, V. Mácha and Š. Nečasovón a system modelling the motion of a piston in a cylinder filled by a viscous heat conducting gas. The piston is moving longitudinally without friction under the influence of the forces exerted by the gas. The fact that the piston is supposed be thermally insulating (adiabatic piston) raises several challenges which received a considerable attention, essentially in the statistical physics literature, during the last decades (see, for instance, [3], [4], [5], [7] and references therein). As far as we know, the closest question which has been studied using the equations of continuum mechanics is the similar problem with a perfectly conducting piston, see Shelukhin [8]. We consider a model based on the Navier-Stokes-Fourier equations in one space dimension for the gas coupled with Newton's law for the piton. The position of the piston being one of the unknowns, we have a free boundary problem. Our main results assert the global in time existence of strong solutions and that the state trajectories converge to an equilibrium state when $t \to \infty$.

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