

On weighted mixed-norm Sobolev estimates for some basic parabolic equations

Li Ping, Pablo R. Stinga¹, José L. Torrea²

SUMMARY

Novel global weighted parabolic Sobolev estimates, weighted mixed-norm estimates and a.e. convergence results of singular integrals for evolution equations are obtained. Our results include the classical heat equation, the harmonic oscillator evolution equation

$$\partial_t u = \Delta u - |x|^2 u + f,$$

and their corresponding Cauchy problems. We also show weighted mixed-norm estimates for solutions to degenerate parabolic extension problems arising in connection with the fractional space-time nonlocal equations $(\partial_t - \Delta)^s u = f$ and $(\partial_t - \Delta + |x|^2)^s u = f$, for $0 < s < 1$.

Keywords: Heat equation, harmonic oscillator evolution equation, degenerate parabolic extension problem.

AMS Classification: 35K10, 35B45, 42B37

References

- [1] N. V. KRYLOV. *Lectures on Elliptic and Parabolic Equations in Hölder Spaces*, Graduate Studies in Mathematics **12**, American Mathematical Society, Providence, R.I., 1996.
- [2] N. V. KRYLOV The Calderón-Zygmund theorem and its applications to parabolic equations. (Russian) *Algebra i Analiz* **13** (2001), 1–25; translation in *St. Petersburg Math. J.* **13** (2002), 509–526.
- [3] P. R. STINGA AND J. L. TORREA. Extension problem and Harnacks inequality for some fractional operators. *Comm. Partial Differential Equations* **35** (2010), 2092–2122.

School of Mathematics and Statistics,
Wuhan University,
430072 Wuhan, China and
Departamento de Matemáticas
Universidad Autónoma de Madrid
28049 Madrid, Spain email: marrisa0708@163.com

¹Department of Mathematics
Iowa State University
396 Carver Hall, Ames, IA 50011, United States of America email:
stinga@iastate.edu

²Departamento de Matemáticas
Universidad Autónoma de Madrid
28049 Madrid. email: joseluis.torrea@uam.es