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Sparse exponentials and monomials, superresolution and an old trick

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SUMMARY

Reconstructing a function of the form

$$f(x) = \sum_{\omega \in \Omega} f_\omega \, e^{\omega \cdot x}$$

from samples on integer grids is known as *Prony's problem*, solved in 1795 [2]. The important point is that the set Ω of complex frequencies is *sparse*, i.e., consists of only a few elements. Also the problem of recovering *sparse polynomials*

$$f(x) = \sum_{\alpha \in A} f_\alpha \, x^\alpha,$$

can be reduced to to solving Prony's problem. In one variable, this question has been considered numerically in the context of radar measurements [3], more recently it has been related to superresolution, but generally there are plenty of related mathematical questions, see [1].

The talk points out the algebraic structure of this problem in *several variables* and how an approximate solution of this problem can be obtained very fast by methods from numerical Linear Algebra as well as by purely symbolic methods. In addition, some aspects of minimal sampling set, in particular their dependency on the set Ω, will be discussed.

Keywords: Sparse recovery, ideal bases, superresolution

AMS Classification: 65F30, 13P10, 13P15

References

[1] G. Plonka and M. Tasche, *Prony methods for recovery of structured functions*, GAMM–Mitt. **37** (2014), 239–258.

[2] C. Prony, *Essai expérimental et analytique sur les lois de la dilabilité des fluides élastiques, et sur celles de la force expansive de la vapeur de l'eau et de la vapeur de l'alkool, à différentes températures*, J. de l'École polytechnique **2** (1795), 24–77.

[3] R. Roy and Th. Kailath, *ESPRIT – estimation of signal parameters via rotational invariance techniques*, IEEE Trans. Acoustics, Speech and Signal Processing **37** (1989), 984–995.

[4] T. Sauer, *Prony's method in several variables*, Submitted for publication (2015), arXiv:1602.02352.

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