

Analytic bifurcation for singular problems

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SUMMARY

Let Ω be a domain in \mathbb{R}^N , $N \geq 2$, $\lambda > 0$ a bifurcation parameter and $f : \mathbb{R} \rightarrow \mathbb{R}$ a “real analytic” type map such that $f(t)$ has superlinear growth as $t \rightarrow \infty$. We consider semilinear elliptic PDEs with the presence of a strong singular term as below:

$$(P_\lambda) \quad \begin{cases} -\Delta u = \lambda(u^{-\delta} + f(u)) & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \quad u|_{\partial\Omega} = 0. \end{cases}$$

Here the singular exponent δ is allowed to be any positive number. We are interested in this work to analyse the problem (P_λ) using the framework of analytic bifurcation theory as developed in the works of [BUFFONI, DANCER, TOLAND, [1] and [2]]. We obtain an analytic global unbounded path of solutions to (P_λ) for any $\delta > 0$ using this framework. In two dimensions when $0 < \delta < 1$ and for certain classes of nonlinearities f that have critical growth (in the sense of Trudinger-Moser imbedding), we show the existence of an analytic unbounded path of solutions to (P_λ) whose Morse index is unbounded along this path. As a consequence, this path admits infinitely many “turning points”.

Keywords: Global analytic bifurcation theory, Morse index, Asymptotic analysis, Turning points, ...

AMS Classification: 35J65, 35J20, 35J70

References

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