Parameter-uniform numerical methods for singularly perturbed problems

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SUMMARY

Singularly perturbed differential equations are often characterized by the presence of a small parameter multiplying the highest order derivative term(s) in the differential equation. The solutions typically exhibit steep gradients in narrow regions (often called layers) of the domain [1, 3]. Given the presence of layers, global (as opposed to simply nodal) accuracy is required. In estimating the accuracy of any proposed approximate solutions, it is important to discuss the choice of norm used to measure such accuracy. Standard numerical methods typically fail to accurately capture these layers. Parameter-uniform numerical methods are designed to generate pointwise accurate numerical approximations to the continuous solution throughout the entire domain and to the appropriately scaled derivatives of the solution. This talk will outline the rationale for the definition of parameter-uniform numerical methods and it will also highlight some of the central tools used in the numerical analysis associated with these singularly perturbed problems [2, 4].

Keywords: Singularly perturbed, parameter uniform, Shishkin mesh

AMS Classification: 65L11, 65L12, 65M06, 65M12

References

- [1] P.A. Farrell, A.F. Hegarty, J.J.H. Miller, E. O'Riordan, G.I. Shishkin, *Robust computational techniques for boundary layers*, Chapman and Hall/CRC Press, Boca Raton, U.S.A., 2000.
- [2] J.J.H. Miller, E. O'Riordan and G.I. Shishkin, *Fitted Numerical Methods for Singular Perturbation Problems*, World-Scientific, Singapore (Revised edition), 2012.
- [3] H.-G. Roos, M. Stynes and L. Tobiska, Robust numerical methods for singularly perturbed differential equations, Springer-Verlag, Berlin Heidelberg, 2008.
- [4] G.I. Shishkin and L.P. Shishkina, Discrete Approximation of Singular Perturbation Problems, Chapman and Hall/CRC Press, Boca Raton, U.S.A., 2008.

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