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Numerical integration under certain type of convexity

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SUMMARY

We propose a method to approximate the integral of a given real-valued function of multiple variables by cubature formulas (numerical integration), which approximate from above (or from below) all functions having a certain type of convexity. We first present the most relevant results for such curbature formulas and establish a characterization result between them and the partition of unity of the integration domain. Moreover, we show that the error estimates based on such cubature formulas are always controlled by the Lipschitz constants of the gradients, the different types of convexity and the error associated with using the quadratic function. In addition, knowing that the function satisfies the classical convexity or strong convexity, we establish sharp upper as well as lower refined bounds for the error estimates. As an application, we define and study a class of cubature formulas on an arbitrary polytope, which approximate strongly convex functions from above. Finally, we present numerical examples illustrating the proposed method.

Keywords: Cubature formulas, Positive definite formulas, Negative definite formulas, Strongly convex functions, Partition of unity.

AMS Classification: 65D32

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