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An alternating direction method for solving 2D convection-diffusion problems with time dependent boundary conditions†

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SUMMARY

In this talk we develop and analyze an efficient uniformly convergent numerical method, of type alternating directions, for solving singularly perturbed initial/boundary value problems of the following type:

Find $u(x, y, t) : \bar{\Omega} \times [0, T] \subset \mathcal{R}^3 \to \mathcal{R}$ solution of

$$\left\{ \begin{array}{l} \dfrac{\partial u}{\partial t} - \varepsilon \Delta u + \overrightarrow{v}(x, y, t).\overrightarrow{\nabla} u + k(x, y, t)u = f(x, y, t), \text{ in } \Omega \times (0, T], \\ u(x, y, 0) = \varphi(x, y), \text{ in } \Omega, \\ u(x, y, t) = g(x, y, t). \text{ in } \partial\Omega \times [0, T]. \end{array} \right. \quad (1)$$

Here, we choose $\Omega \equiv [0, 1] \times [0, 1]$ and we assume that the convective term satisfies $v_i(x, y, t) \geq \alpha > 0, \ i = 1, 2$. We focus our attention on the singularly perturbed case ($\varepsilon << \alpha$) where, in general, regular boundary layers (and also a corner layer) appear in the outflow boundary. In this case, fitted space discretization techniques are necessary to capture successfully the rapid variations of their solutions at these regions. We will also pay special attention to the order reduction phenomenon, which is typical when using classical one step time integrators, and particularly severe if alternating direction schemes are chosen to integrate in time, together with classical evaluations of the boundary conditions. We propose a simple modification for these evaluations, easy to implement, and we analyze it in detail, proving that the order reduction is avoided completely. The technique for the analysis of the uniform convergence of our proposal is innovative too, permitting to remove some spurious restrictions between the time step and the mesh size which were considered in previous papers to complete the analysis of the uniform convergence. Finally, some numerical experiments are shown in order to illustrate the improvements which the method provides.

Keywords: convection-diffusion, fractional Euler method, piecewise uniform meshes, uniform convergence, order reduction

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