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Optimal spline spaces for L2 n-widths

Michael Floater, **Espen Sande**[1]

SUMMARY

Let $X = (X, \|\cdot\|)$ be a normed linear space, A a subset of X, and X_n an n-dimensional subspace of X. Then the Kolmogorov n-width of A relative to X is

$$d_n(A; X) = \inf_{X_n} E(A, X_n; X),$$

where

$$E(A, X_n; X) = \sup_{f \in A} \inf_{g \in X_n} \|f - g\|.$$

A subspace X_n is *optimal* for A provided that $d_n(A; X) = E(A, X_n; X)$.

Kolmogorov found the exact value of the n-width for

$$A^r = \{f : f^{(r-1)} \text{ abs. cont. on } (0,1), \|f^{(r)}\| \leq 1\},$$

where $\|\cdot\|$ is the L^2 norm on $[0, 1]$ and $X = L^2[0, 1]$, and showed that an optimal subspace is the span of the first n eigenfunctions of a boundary value problem.

Later, Melkman and Micchelli [2] showed that A^r also admits two optimal spline subspaces, one of degree $r - 1$, the other of degree $2r - 1$. In this talk we review the analysis of Melkman and Micchelli, in which the concept of totally positive kernels plays a crucial role. We then discuss whether spline spaces of other degrees might also be optimal, as suggested by numerical tests carried out by Evans et al. [1], which would be of interest in isogeometric analysis.

Keywords: n-width, spline space, total positivity, isogeometric analysis

AMS Classification: 41A15, 65D07, 65N30

References

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[1]Department of Mathematics, University of Oslo, Norway email:
michaelf@math.uio.no, espsand@math.uio.no