Non-local fractional derivatives. Discrete and continuous

Luciano Abadías[1], **Marta de León-Contreras**[2], **José Luis Torrea**[2]

SUMMARY

We prove maximum and comparison principles for fractional discrete derivatives in the integers. Regularity results when the space is a mesh of length h, and approximation theorems to the continuous fractional derivatives are shown. When the functions are good enough, these approximation procedures give a measure of the order of approximation. These results also allows us to prove the coincidence, for good enough functions, of the Marchaud and Grünwald-Letnikov derivatives in every point and the speed of convergence to the Grünwald-Letnikov derivative. The fractional discrete derivative will be also described as a Neumann-Dirichlet operator defined by a semi-discrete extension problem. Some operators related to the Harmonic Analysis associated to the discrete derivative will be also considered, in particular their behavior in the Lebesgue spaces $\ell^p(\mathbb{Z})$.

[1]Departamento de Matemáticas, Instituto Universitario de Matemáticas y Aplicaciones, Universidad de Zaragoza, 50009 Zaragoza, Spain. email: `labadias@unizar.es`

[2]Departamento de Matemáticas
Universidad Autónoma de Madrid, 28049 Madrid, Spain.
email: `marta.leon@uam.es`

[2]Departamento de Matemáticas
Universidad Autónoma de Madrid and Instituto de Ciencias Matemáticas (CSIC-UAM-UC3M-UCM), Spain.
email: `joseluis.torrea@uam.es`