ON ABSTRACT INDEFINITE CONCAVE-CONVEX PROBLEMS AND APPLICATIONS TO QUASILINEAR ELLIPTIC EQUATIONS

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In this work we present several existence results on critical values for an abstract concave-convex problem involving 3 functional having different homogeneities. The critical points are obtained by minimization on subsets of the related Nehari manifold associated to the functional. Then we apply these results to various quasilinear elliptic problems, as for instance, the following p-laplacian concave-convex **Problem I** with Steklov boundary conditions on a bounded regular domain

$$\left\{\begin{array}{rcll} -\Delta_p u + V(x)u^{p-1} & = & 0 & \text{in } \Omega; \\ |\nabla u|^{p-2}\dfrac{\partial u}{\partial \nu} & = & \lambda a(x)u^r + b(x)u^q & \text{on } \partial\Omega; \\ u > 0 \text{ in } \Omega, & & & \end{array}\right.$$

with given functions a, b, V possibly indefinite and $1 < r < p < q$. We first generalize partially to nonlinear operators the results of [1, 2, 3, 4, 5] about critical points in the Nehari manifold. Secondly we prove up to 4 solution of **Problem II**:

$$\left\{\begin{array}{rcll} -\Delta_p u + V(x)u^{p-1} & = & \lambda a(x)u^{r-1} & \text{in } \Omega; \\ |\nabla u|^{p-2}\dfrac{\partial u}{\partial \nu} & = & b(x)u^{q-1} & \text{on } \partial\Omega; \\ u > 0 \text{ in } \overline{\Omega}. & & & \end{array}\right. \tag{1}$$

and **Problem III**:

$$\left\{\begin{array}{rcll} \Delta_p^2 u - c|u|^{p-2}u & = & \lambda a(x)|u|^{r-2}u + b(x)u^{q-2}u & \text{in } \Omega; \\ u = \Delta u = 0 & & & \text{on } \partial\Omega, \end{array}\right. \tag{2}$$

with $c \in \mathbb{R}$.

By putting the former problem in an abstract setting we are able to apply our abstract existence results to other boundary problems with either different boundary conditions (c.f Problem II) or different differential operator, as the p-bilaplacian (c.f. Problem III).

References

[1] A. Ambrosetti ,H. Brézis, G. Cerami, Combined effect of concave and convex nonlinearities in some elliptic problems, J. Funct. Anal. 122, (1994), 519-543.

[2] K.J. Brown, The Nehari manifold for a semilinear elliptic equation involving a sublinear term, Calc. Var. Partial Differ. Equa. 22(4), (2005), 483-494.

[3] K. J. Brown, Tsung-Fang Wu, A fibering map approach to a semilinear elliptic boundary value problem, Electronic Journal of Differential Equations,, vol. 2007, No 69, (2007), 1–9.

[4] K.J. Brown, Y. Zhang, The Nehari manifold for a semilinear elliptic equation with a sign-changing weight function, J. Differ. Equ. 193(2), (2003), 481-499.

[5] T.F. Wu, Multiplicity of positive solution of p-Laplacian problems with sign-changing weight functions, Int. J. Math. Anal. (Ruse) 1 (912), (2007), 557-563.

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