On the Heston model in mathematical finance: An analytic approach by PDEs.

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SUMMARY

We study the Heston model for pricing European options on stocks with stochastic volatility. This is a Black-Scholes-type equation whose spatial domain is the half-plane $\mathbb{H} = \mathbb{R} \times (0, \infty)$; the equation is parabolic and degenerates at the boundary $\partial\mathbb{H} = \mathbb{R} \times \{0\}$. However, we are able to show that the Black-Scholes-type operator, which appears in this equation, generates a holomorphic C^0-semigroup in a suitable weighted L^2-space over $\mathbb{H}$. We show that the C^0-semigroup solution can be extended to a holomorphic function in a complex domain.

Keywords: Heston model; stochastic volatility, Black-Scholes equation; European call option, degenerate parabolic equation; terminal value problem, holomorphic extension; analytic solution.

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