

Existence of martingale solutions to a pseudomonotone evolution equation with multiplicative noise

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SUMMARY

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a classical Wiener space endowed with a filtration $(\mathcal{F}_t)_{t \in [0, T]}$, $T > 0$ with the usual assumptions, $D \subset \mathbb{R}^d$ be a bounded Lipschitz domain, $Q := (0, T) \times D$ and $p > 2$. Our aim is the study of the problem

$$(P) \begin{cases} du - \operatorname{div}(|\nabla u|^{p-2} \nabla u + F(u)) dt = H(u) dW & \text{in } \Omega \times (0, T) \times D \\ u = 0 & \text{on } \Omega \times (0, T) \times \partial D \\ u(0, \cdot) = u_0 & \text{in } \Omega \times D \end{cases}$$

for a cylindrical Wiener process in $L^2(D)$ and $F : \mathbb{R} \rightarrow \mathbb{R}^d$ Lipschitz continuous. We consider the case of multiplicative noise with $H : L^2(D) \rightarrow HS(L^2(D))$, $HS(L^2(D))$ being the space of Hilbert-Schmidt operators, satisfying appropriate regularity conditions. By an implicit time discretization of (P) , we obtain approximate solutions. Using the theorems of Skorokhod and Prokhorov, we are able to pass to the limit and show existence of martingale solutions.

Keywords: pseudomonotone problem, stochastic forcing, cylindrical Wiener process, martingale solution

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